A Parametric Level Set Approach for Dual Energy Computerized Tomography

Oguz Semerci and Eric L. Miller, Tufts University

Contact: *oguz.semerci@tufts.edu, elmiller@ece.tufts.edu*

Abstract

The purpose of this work is to introduce a novel polychromatic dual energy algorithm with an emphasis on detection of objects of interest, whose chemical properties are assumed to be known with some level of uncertainty. These objects are modeled as piecewise constant regions embedded in unknown background. We determine the shape, number (perhaps zero) and material properties of these regions along with images of the background. As such, we provide a unified approach to object detection, characterization, and image formation. In more detail, the reconstruction scheme is a two step non-linear optimization process. In the first step, a level-set approach is used to determine the boundary of the object. Using the results from the first step, the second step conducts a pixel-wise reconstruction of photoelectric effect coefficient and Compton scattering coefficient of background pixels. This stage of processing is cast as a regularized non-linear inverse problem. Due to a severe mismatch in the sensitivity of the data to the two physical parameters of interest (photoelectric and Compton coefficients), we introduce a new gradient-based similarity regularizer to obtain accurate reconstruction of the background properties. Numerical results show that the algorithm successfully detects objects of interest, find their shape and location, and gives an accurate reconstruction of the background.

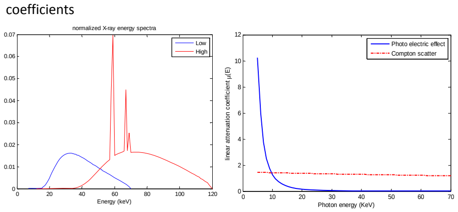
ALERT
Three-Level Strategic Approach

State of Art

- Dual energy x-ray tomography allows material characterization[1].
- Sinogram estimation(noise sensitive process) followed by a FBP algorithm [1,4].
- Statistical reconstruction algorithms[5].
- Pre-built look up tables to efficiently obtain basis material decompositions [2].

Dual Energy Spectra

- Photoelectric effect is not significant for $E > 30\text{KeV}$
- Whereas, Compton scatter is almost constant throughout the spectrum
- Use different energy spectra to obtain *Compton* and *photoelectric* coefficients

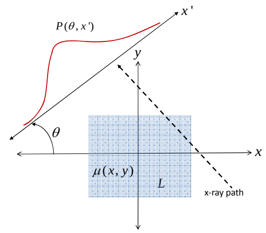


Polychromatic CT Formulation

$$P(\theta,x') = -\ln\left[\int S(E)e^{-M(\theta,x')}\mathrm{d}E\right] + \ln\int S(E)\mathrm{d}E$$

$S(E)$: X-ray energy spectrum

$M(\theta,x')$: Radon transform of the linear attenuation coefficient $\mu(x,y)$



$$M(\theta,x') = \int_L \mu(x,y)\delta(x\cos\theta + y\sin\theta - x')\mathrm{d}x\mathrm{d}y$$

$$\mu(x,y) = f_{KN}(E)c(x,y) + f_p(E)p(x,y)$$

Compton Scatter — Photoelectric Effect

f_{KN}: Klein-Nishina cross section, $f_p = 1/E^3$

- X-ray attenuation is due to Compton scatter Photoelectric effect
- f_{KN} and f_p reflects the dependency of these phenomena on energy
- Energy dependency of attenuation initially investigated by Alvarez&Macovski[1]
- c and p are the Compton and photoelectric coefficients to be determined.

$$c = k_1 Z \qquad p = k_2 Z^n, \; n \approx 3.5 \qquad Z: \text{atomic number}$$

Compton and Photo Electric Images

$$c(x,y) = \chi_\Omega(x,y)c_{object} + [1-\chi_\Omega(x,y)]\sum_i^N B_i(x,y)\beta_{c_i}$$

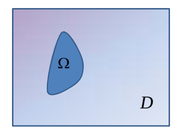
$$p(x,y) = \chi_\Omega(x,y)p_{object} + [1-\chi_\Omega(x,y)]\sum_i^N B_i(x,y)\beta_{p_i}$$

$B_i(x,y)$: basis functions for background

β_{i_p}, β_{i_c}: unknown expansion coefficients

Shape Model for the Object of Interest

- We assume objects of interest with known properties on an unknown background
- For the objects, level set representation is suitable
- A low order basis expansion is used for Level set function as well the background



$$\chi_\Omega(x,y) = H\left(\sum_{i=1}^{N_b} P_i(x,y)\alpha_i\right)$$

$$\chi_\Omega(x,y) = \begin{cases} 1 & ,(x,y)\in\Omega \\ 0 & ,(x,y)\in D\setminus\Omega \end{cases}$$

$\chi_\Omega(x,y)$: characteristic function of the objects

H: step function

$P_i(x,y)$: basis functions for the level set function

α_i: unknown expansion coefficients

Polynomial basis: $P_{N_b} = \{\alpha_{k_1,k_2} = x_1^{k_1}x_2^{k_2} \quad \text{for } k_1,k_2 = 0,\ldots,\sqrt{N_b}-1\}$

Radial basis: $P_{N_b} = \{\alpha_k = \phi(\|r - r_k\|) \quad \text{for } r = [x_1\ x_2]^T,\ k = 0,\ldots,N_b\}$

Reconstruction Algorithm

Cost function with three terms is minimized via Levenberg-Marquardt algorithm :

1. Data miss match term:

$$F_1 = \|K(\mathbf{p}) - \mathbf{m}\|^2_{L_2(D)}$$

K: discrete version of the forward model

$\mathbf{p}$: vector of unknowns

$\mathbf{m}$: vector of measurements

2. Object Classification Term:

- Compton and photoelectric coefficients of the objects of interest are assumed to have a two dimensional Gaussian distribution on the plane with means c_0, p_0 and standard deviations of σ_c *and* σ_p respectively.

$$F_2 = \lambda_1 \int_D \left[\left(\frac{c_{object} - c_0}{\sigma_c}\right)^2 + \left(\frac{p_{object} - p_0}{\sigma_p}\right)^2\right]\chi_\Omega(x_1,x_2)\mathrm{d}x_1\mathrm{d}x_2$$

λ_1: is the tuning parameter for the classification term

3. Regularization Term for Photo Electric Image Reconstruction:

A priori: Compton coefficient image and photoelectric coefficient images should be morphologically alike

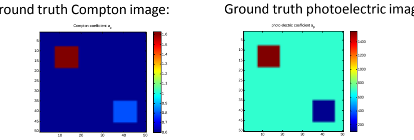
- In order to enforce the structural similarities between two images we introduced a correlation type of metric enforcing the similarity between the gradient vectors.

$$F_3 = \lambda_2 \left(\frac{\|D\beta_c\|^2 \|D\beta_p\|^2}{(D\beta_c)^T(D\beta_p)} - 1\right)^2 \qquad D: \text{gradient matrix}$$

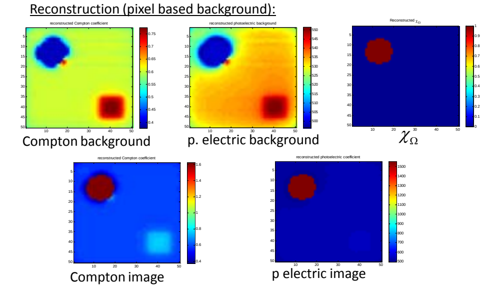
Reconstruction Examples

- An object of interest is located on the upper left corner an 50x50 image.
- 60dB noise is added to simulated data.
- 150 Projections for every 3 degrees between 0˚ and 180˚

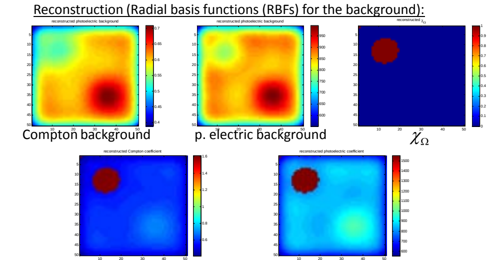
Ground truth Compton image: Ground truth photoelectric image:



Reconstruction (pixel based background):



Reconstruction (Radial basis functions (RBFs) for the background):



References:

[1] R. Alvarez and A. Macovski, "Energy-selective reconstructions in x-ray computerized tomography," *Phys. Med. Biol., vol. 21, no. 5, pp. 733-744, 1976.*

[2]G. Zhang, J. Cheng, L. Zhang, Z. Chen, and Y. Xing, "A practical reconstruction method for dual energy computed tomography," *J. of X-ray Sci. and Tech., vol. 16, no. 2, 2008.*

[3] C. Samson, L. Blanc-Fraud, G. Aubert, and J. Zerubia, "A level set model for image classifcation," *Int. J. Comput. Vis., vol. 40, no. 3, pp. 187-197, 2000.*

[4] Z. Ying, R. Naidu, and C. Crawford, "Dual energy computed tomography for explosive detection," *J. of X-ray Sci. and Tech., vol. 14, no. 4, pp. 235-256, 2006.*

[5] P. Sukovic, N. H. Clinthrone, J.A. Fessler, N. Elbakri, "Maximum-likelihood dual-energy tomographic image reconstruction," *SPIE Med. Imag., vol. 4684, no. 2, pp. 38-49, 2002.*

Acknowledgements:

This material is based upon work supported by the U.S. Department of Homeland Security under Award Number 2008-ST-061-ED0001. The views and conclusions contained in this document are those of the authors and should not be interpreted as necessarily representing the official policies, either expressed or implied, of the U.S. Department of Homeland Security.