Sinogram restoration for computed tomography

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Conclusions

- Sinogram restoration is a statistically principled way of preprocessing CT data prior to reconstruction by analytic algorithms
- It can not only be used to reduce artifacts (the point of most preprocessing) but also to reduce noise and model geometric effects, as in fully iterative algorithms.
- At the very least, it represents a computationally efficient middle ground between analytic algorithms and fully iterative ones.
  - However, in many studies we have done it performs as well as fully iterative algorithms.
  - It can readily accommodate the proper “nonlinear” model for CT. Many/Most fully iterative algorithms work with linearized system models.
- Naturally it has limitations: harder to accommodate non-negativity and other constraints, harder to implement edge-preserving priors.

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Outline

- Overview of data degradations in CT
- Development of an imaging model
- Introduction of our sinogram restoration strategy
- Results of a collaboration with Philips
- Deeper connections between iterative sinogram-domain methods and fully iterative image reconstruction

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Ideal imaging model in CT

- In CT we seek to reconstruct the X-ray attenuation map from measurements of X-ray transmission along a number of lines.
- Reconstruction algorithms such as filtered backprojection (FBP) assume

$$I_i^{\det} = I_i^{inc} e^{-l_i}$$

where

$$l_i = \int_{L_i} \mu(\vec{x})\, dl$$

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Deviations from the model

- Noise
- Beam hardening
- Scatter
- Off-focal radiation
- Detector speed and afterglow
- Detector crosstalk
- Detector dark current
- Metal artifacts

Noise-induced streak artifacts

At low doses, high noise levels in the most attenuated projections can lead to distracting noise-induced streaks.

No adaptive filtering

Penalized-likelihood sinogram smoothing

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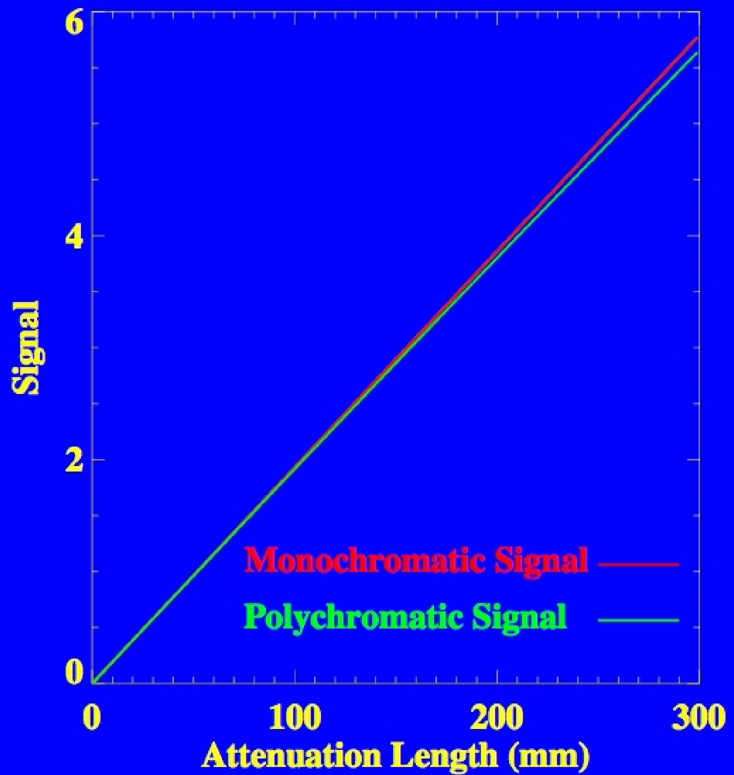
Beam hardening



For water-equivalent materials (i.e., most soft tissues) the effect of beam hardening is a slight non-linearity in the detected signal, which is given by the green line at left rather than the ideal red line.

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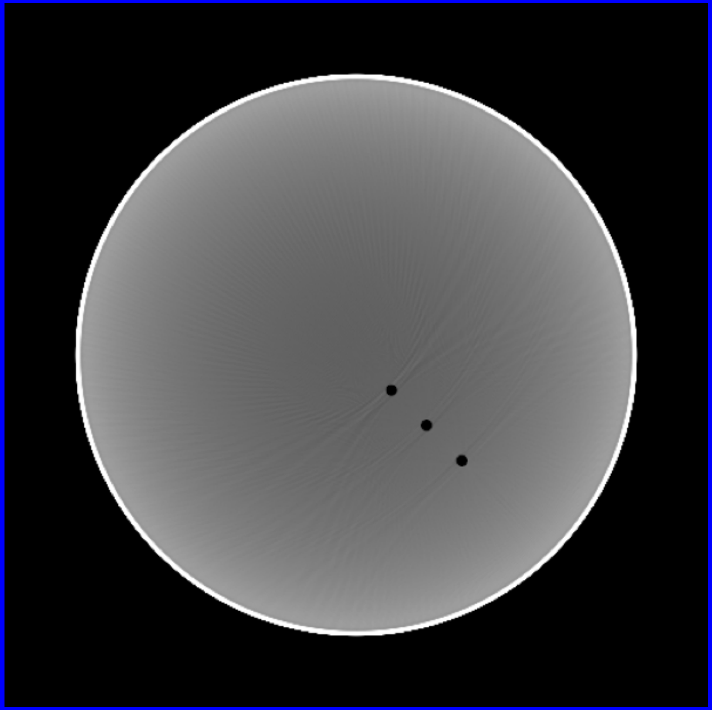
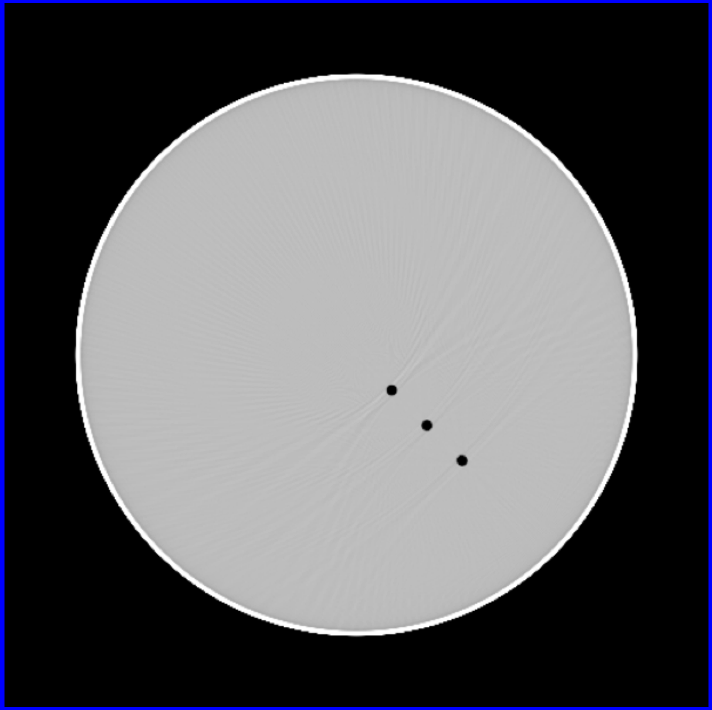
Effect of beam hardening



No correction



Correction by line-integral remapping

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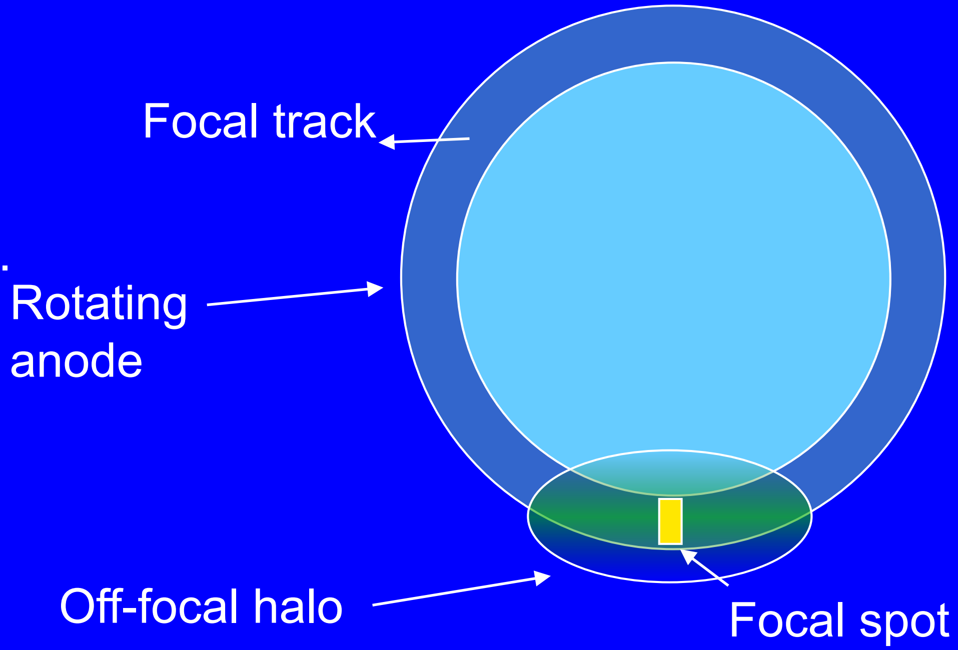
Off-focal radiation

- The vast majority of x-rays emerge from the small focal spot of the tube, but a lower-intensity halo of off-focal x-rays typically surrounds them.
- These arise when secondary electrons escape the focal spot and strike the elsewhere on the anode.
- If not accounted or corrected for, this off-focal radiation can lead to halo-like artifacts at boundaries in reconstructed images.



Schematic of off-focal radiation

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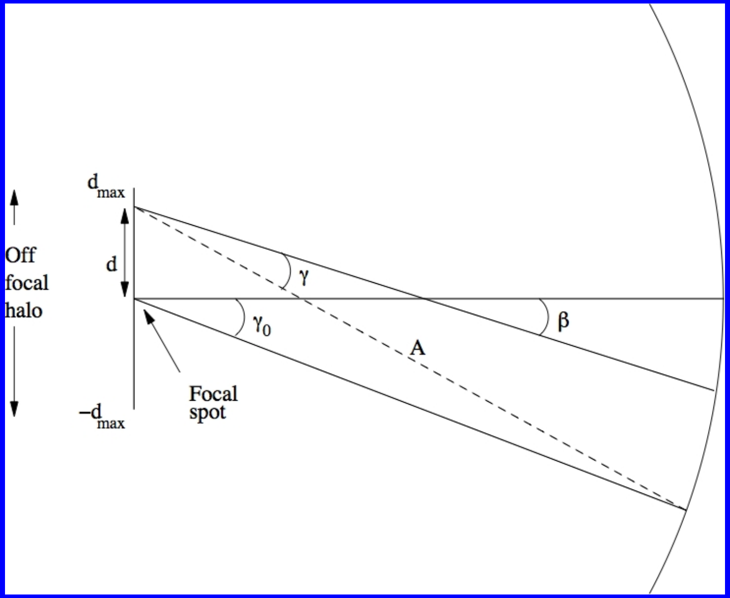
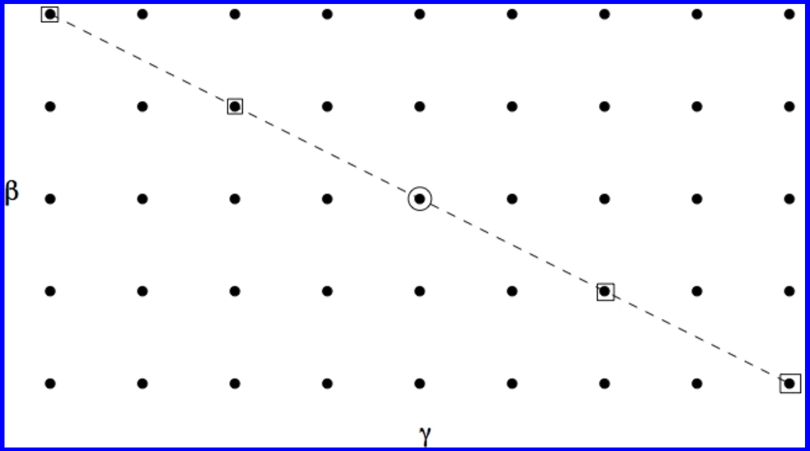
Off-focal radiation

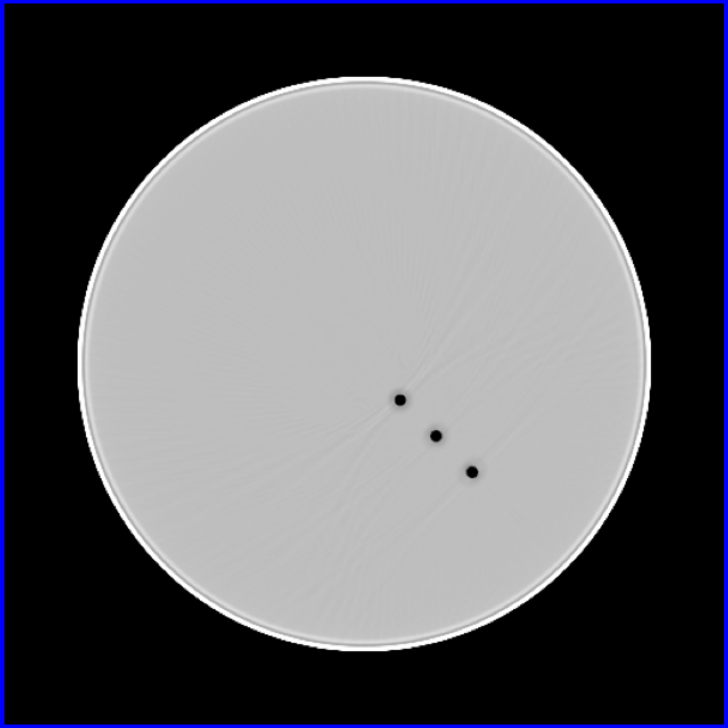


Geometry of off-focal radiation



Off-focal radiation = convolution in sinogram

Effect of off-focal radiation



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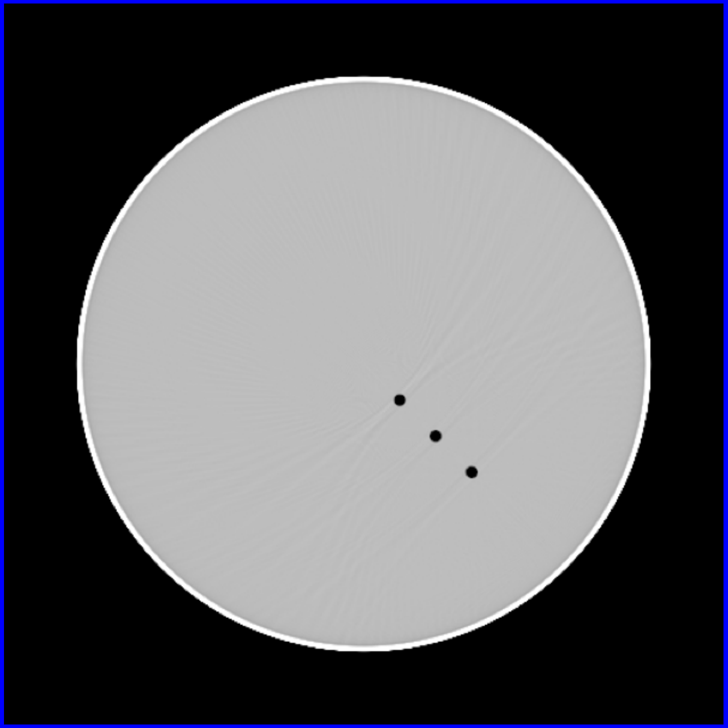

Correction by deconvolution

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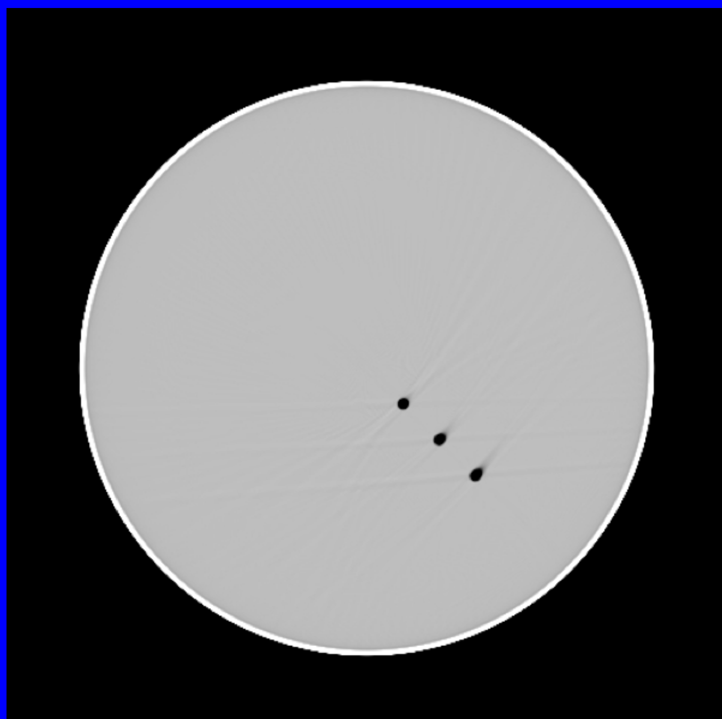
Afterglow

- If detector “lag” is on the order of detector sampling rate, isgnal blurs from one projection to the next.
- Detectors are getting faster but so is sampling rate (flying focal spot, faster gantries, etc).



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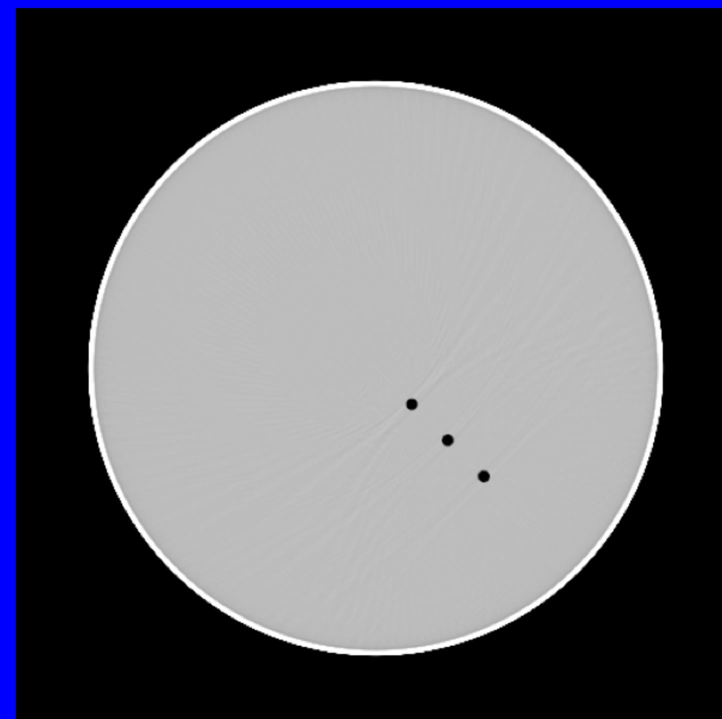
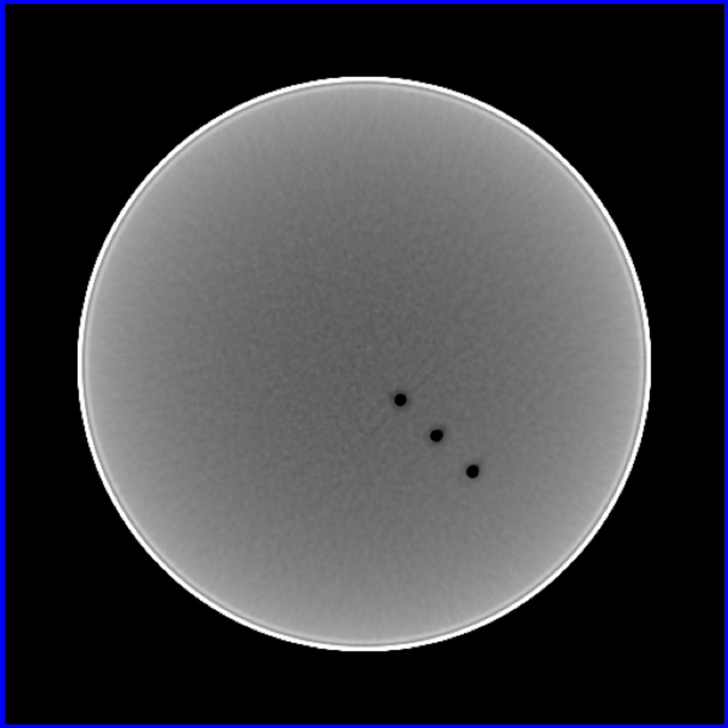


Recursive correction

All the effects



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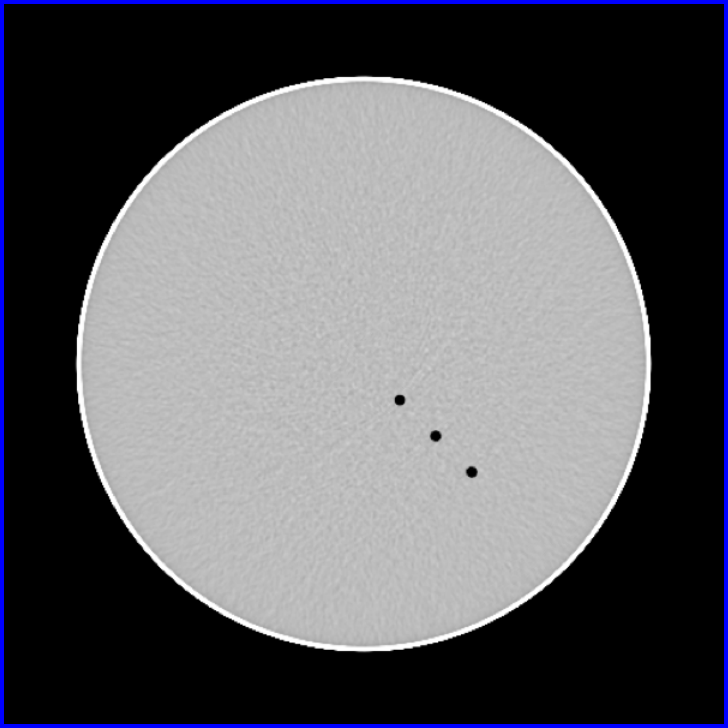

Penalized-likelihood correction

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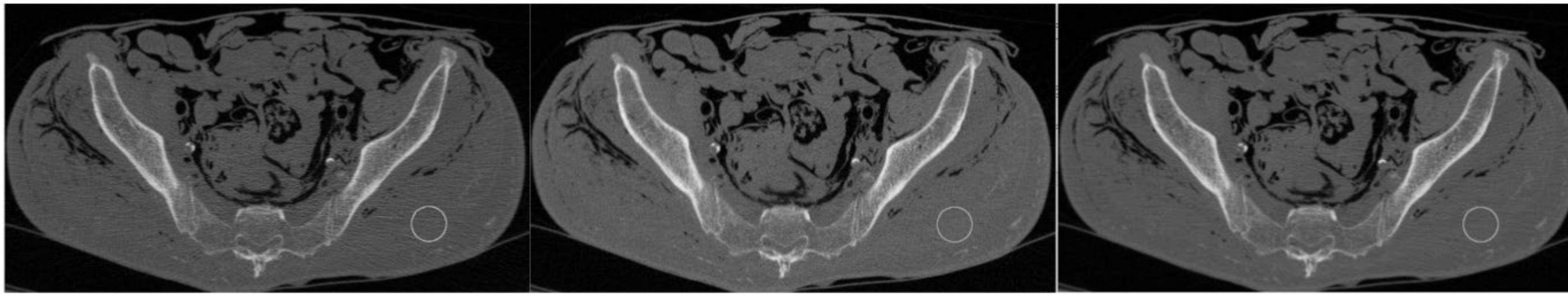
Examples: potential for dose reduction



(a) 35 mA, standard, s. dev. = 153.3

(b) 35 mA, proposed PL, s. dev. = 99.4

(c) 75 mA, standard, s. dev. = 87.15

(d) 35 mA, standard, s. dev. = 241.9

(e) 35 mA, proposed PL, s. dev. = 98.0

(f) 75 mA, standard, s. dev. = 119.3

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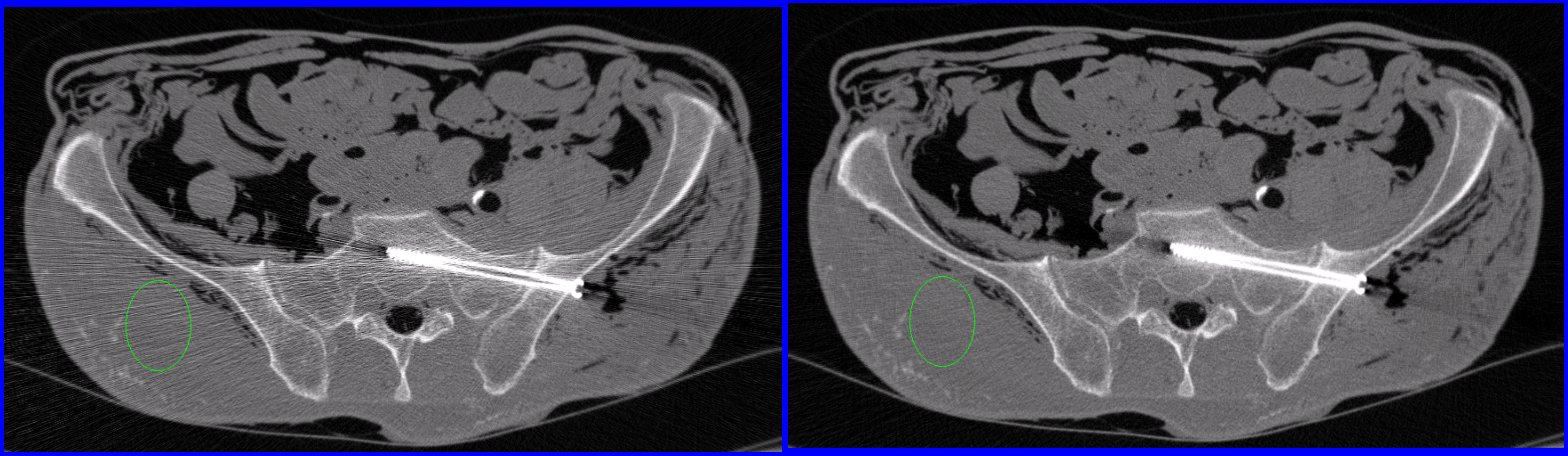
Examples: potential for artifact reduction



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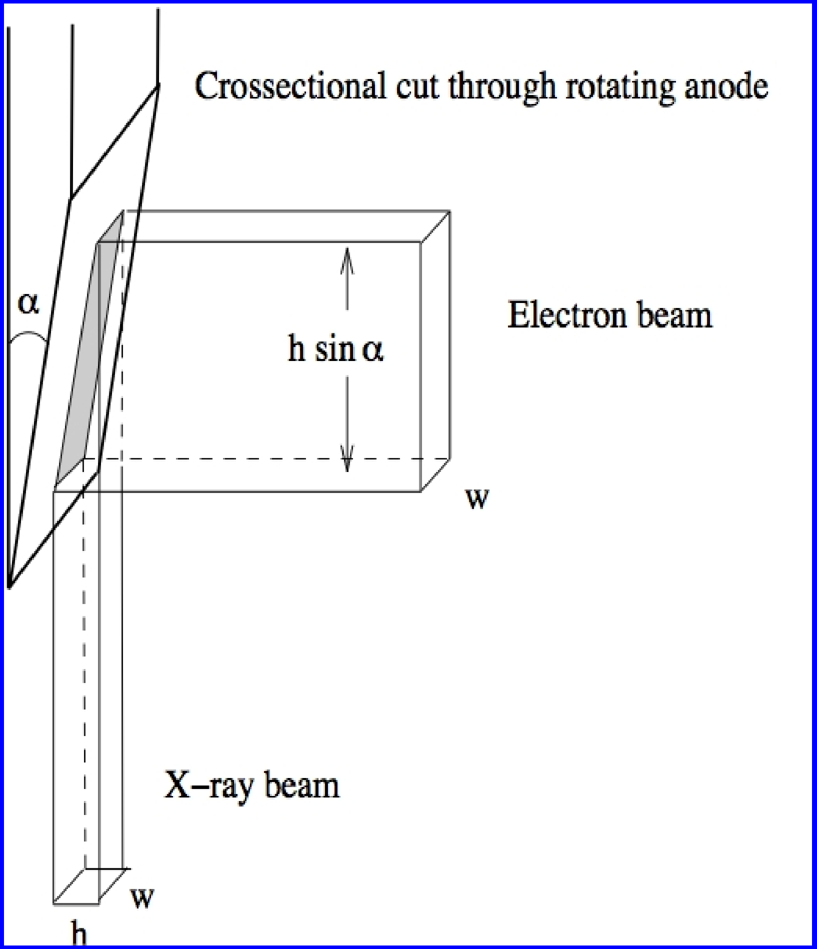
An aside on geometry modeling

- **Heat generation in X-ray tubes is a significant limiting factor.**
  - **To allow heat generated to be spread over a wider area, the anode is rotated and the focal track is also beveled at a shallow angle.**
  - **Thanks to the line focus principle, this allows a fairly large area of the focal track to be exposed to electrons while retaining a fairly small effective projected focal spot.**



$\alpha \sim$ 5-7 degrees in CT tubes

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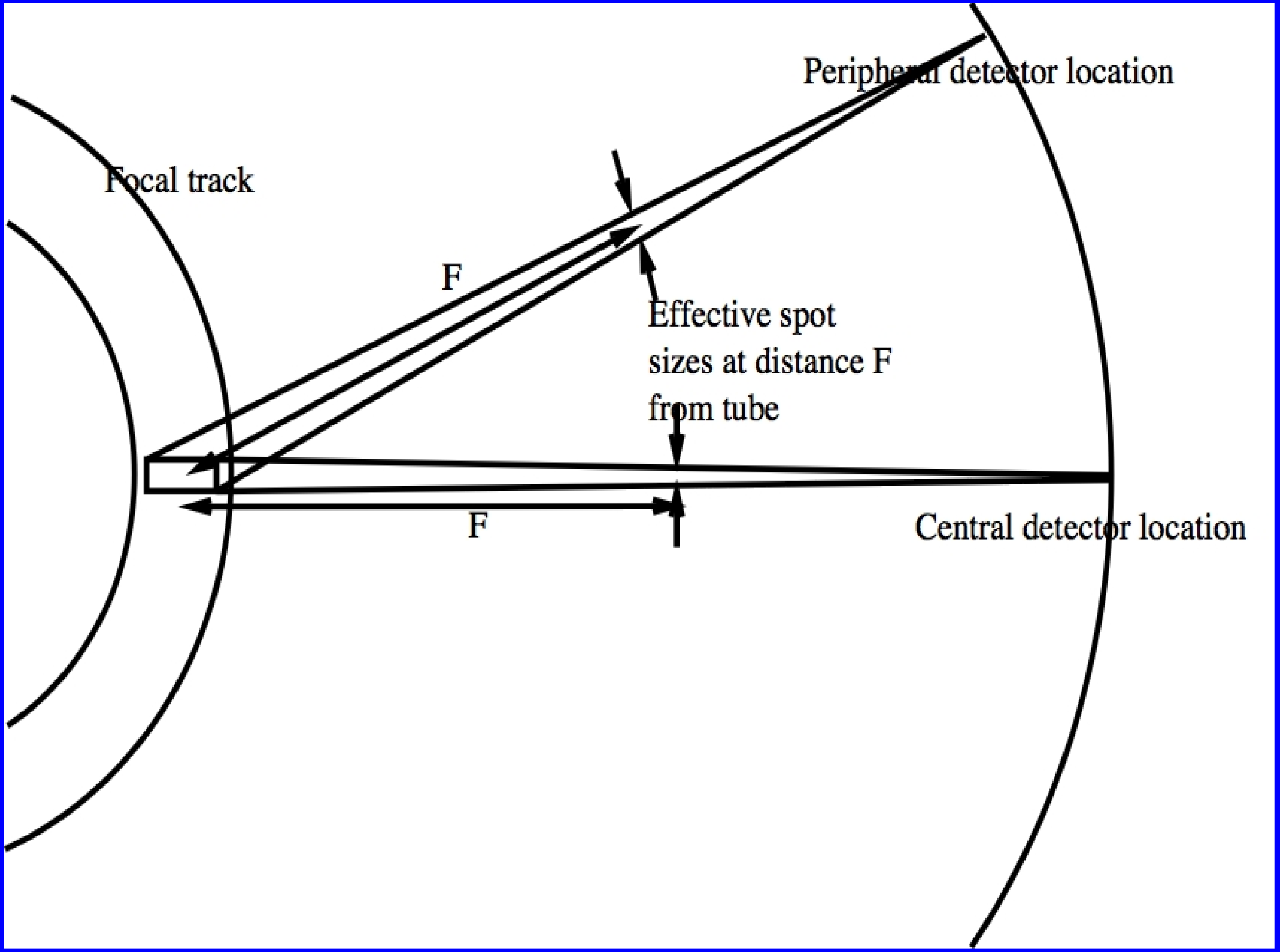
Differential focal spot size

• Focal spot appears larger to peripheral detectors: this leads to spatially varying spatial resolution.

• Our goal is to model and correct for this in the sinogram domain using penalized likelihood sinogram restoration.



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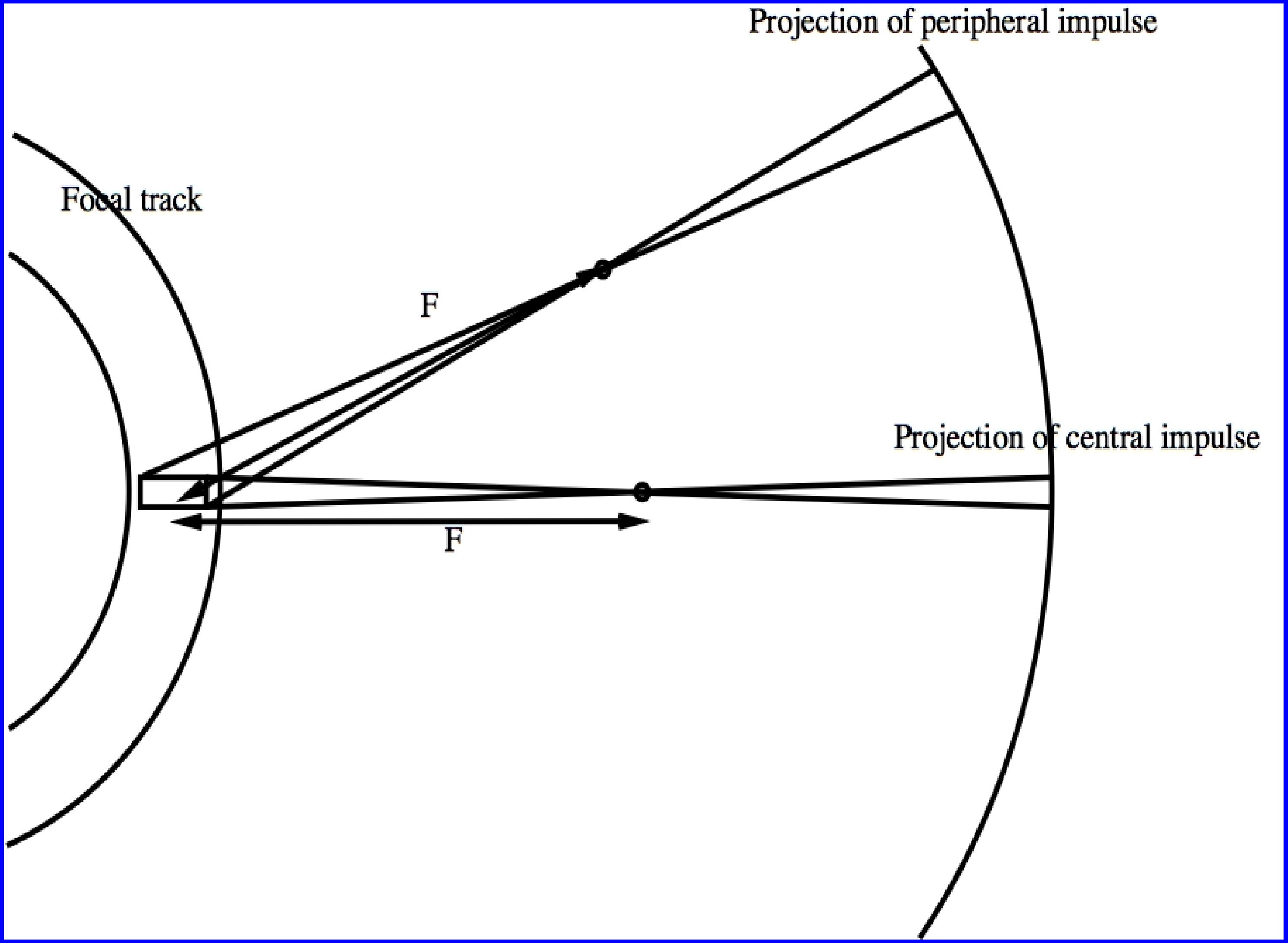
Differential projection of impulse



So a peripheral impulse will produce a broader projection

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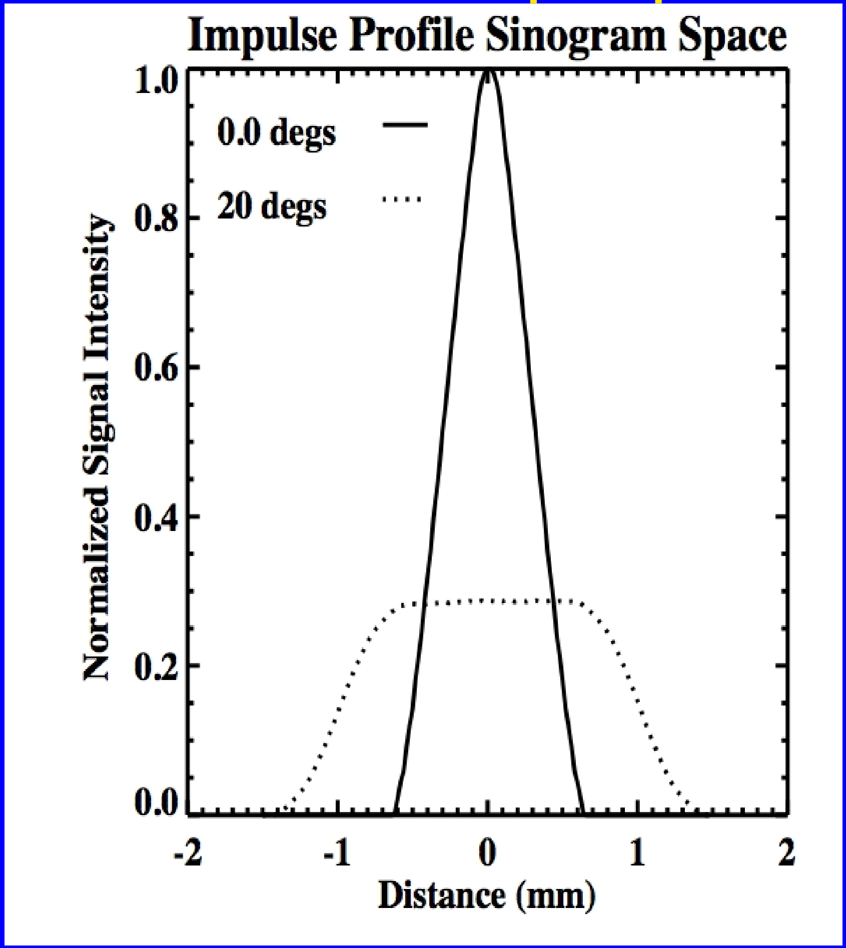
Projection of central and peripheral impulses

Peripheral projection has been superimposed on central projection so they may be compared

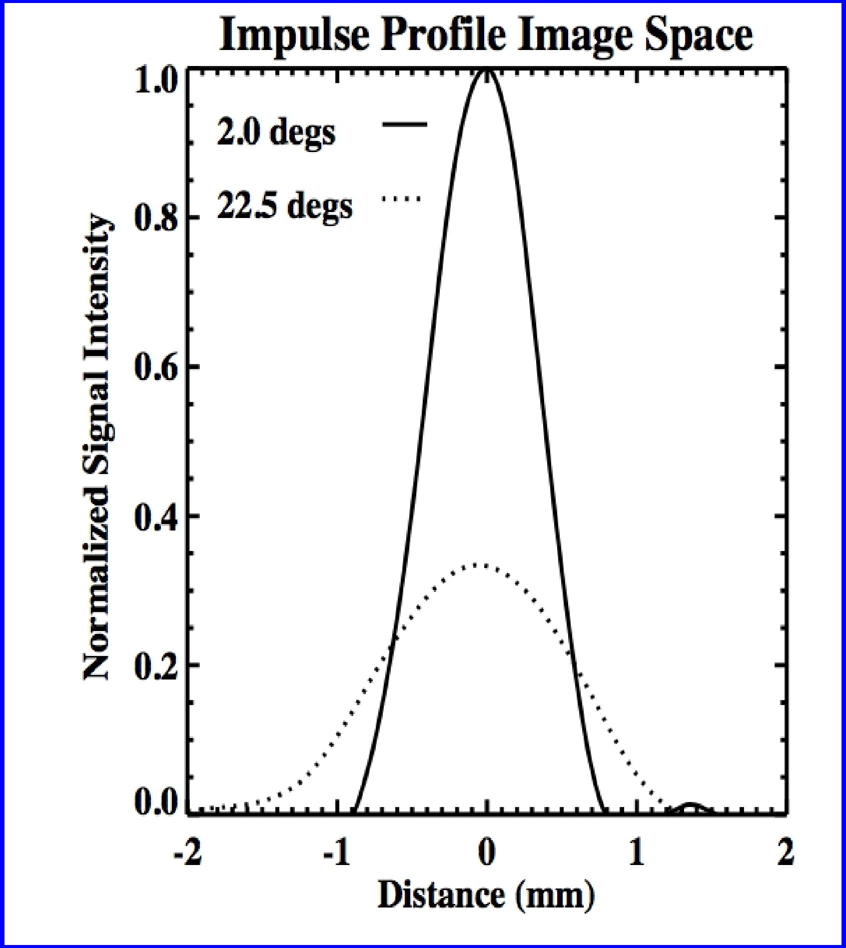


See similar figure in Jiang Hsieh's book *Computed Tomography.*

Peripheral impulse produces a broader projection

Reconstruction of central and peripheral impulses

Profiles through reconstructions of central and peripheral impulses have been superimposed for comparison



Peripheral impulse produces a broader reconstruction

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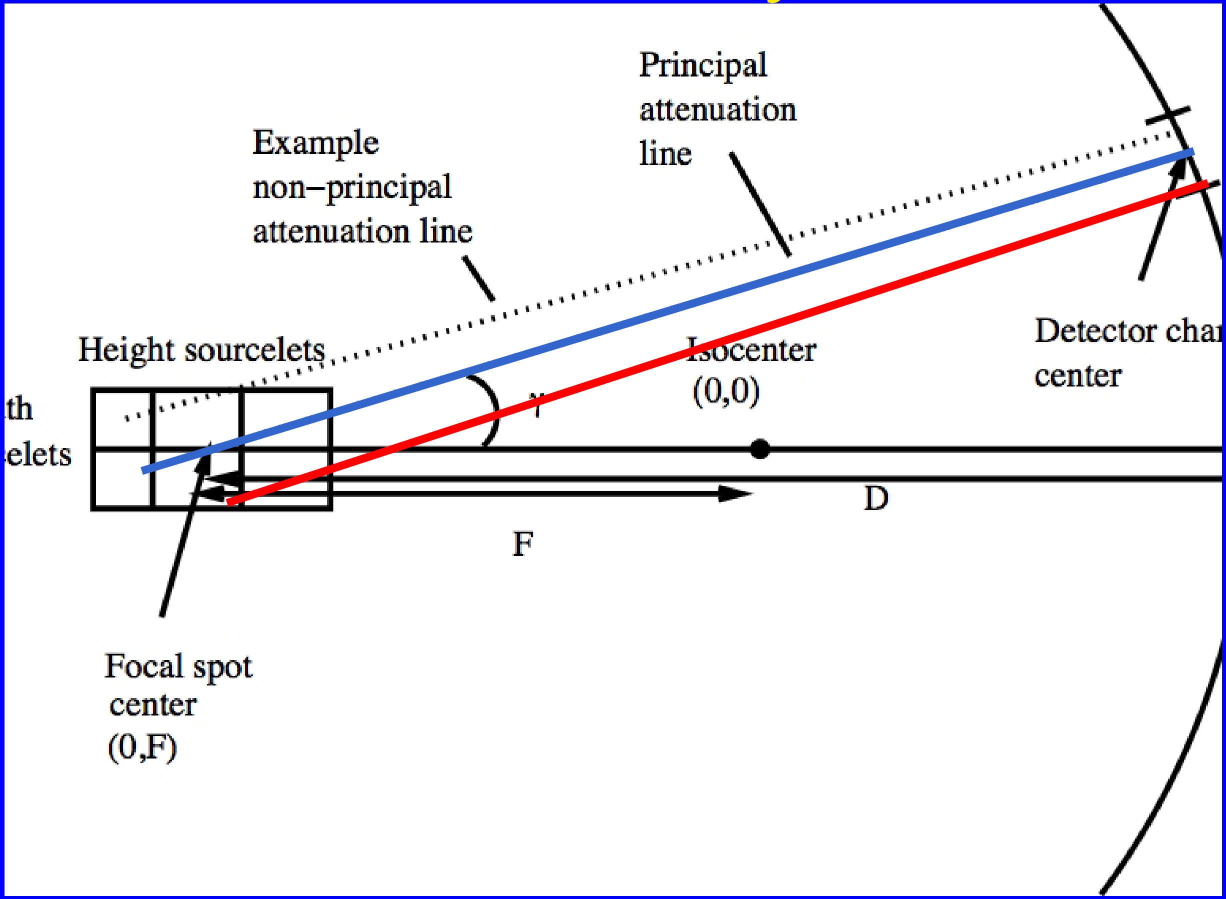
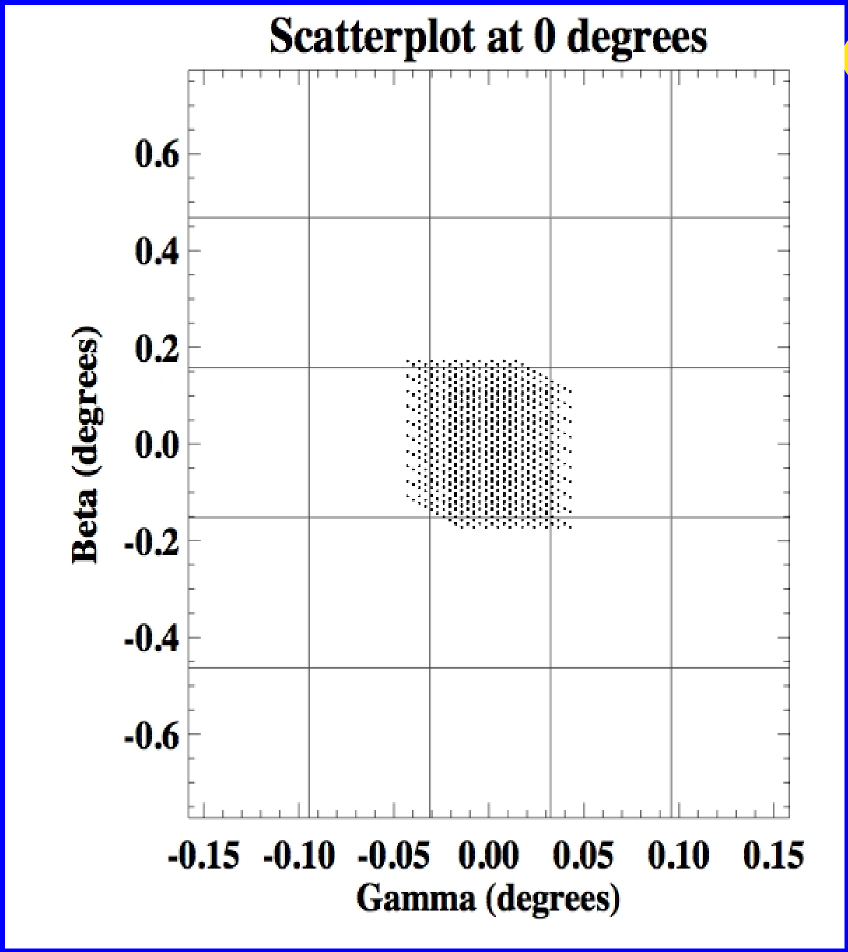
Geometry

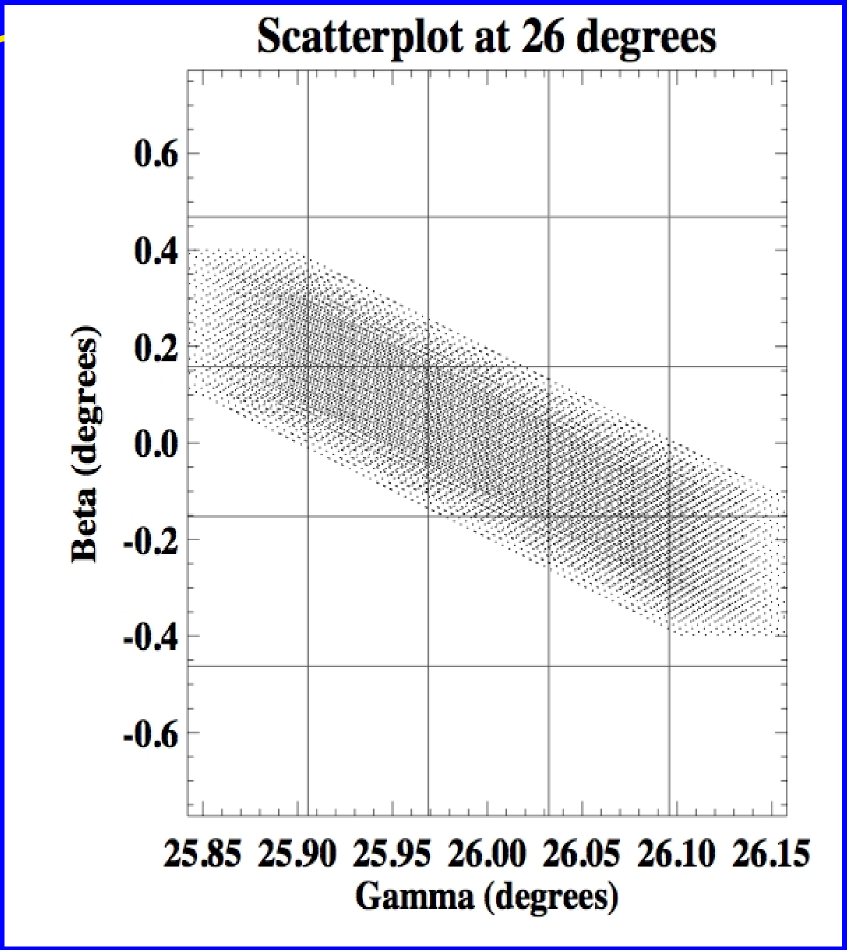


Measurement involves average over (exponentiated) line integrals having a range of different fanbeam coordinates. We can calculate those coordinates and make a scatterplot.

Scatterplot of effective line integral

dir





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Linear vs nonlinear modeling

- This kind of geometric modeling can be represented as

$$y_i = \iint d\gamma\, d\beta\, I(\gamma,\beta) f_i(\gamma,\beta) e^{-\int_{L(\gamma,\beta)} \mu(x,y)\, d\ell}$$

Incident intensity   Effective measurement kernel

- **We can discretize the attenuation map yielding**

$$y_i(\vec{x}) = \sum_j b_{ij} e^{-\sum_k a_{jk} x_k}$$

- **KEY POINT: The coefficients a_{jk} represent intersections of LINES with basis functions.**

Linear vs. nonlinear modeling

As far as I know, no one is working with this equation:

$$y_i\left(\vec{x}\right) = \sum_j b_{ij} e^{-\sum_k a_{jk} x_k}$$

Instead, people use one of three major approaches:

1) The use of raw data but with a single linear approximate matrix:

$$y_i = I_i e^{-\sum_k c_{ik} x_k}$$

2) The use of logged data and a single linear approximate matrix:

$$z_i = -\ln\left(\frac{y_i}{I_i}\right) = \sum_k c_{ik} x_k$$

3) Sinogram restoration, using the first eq. to estimate ideal line integrals, which are then used in FBP:

$$y_i = \sum_j b_{ij} I_j^{(ideal)}, \text{ where } I_j^{(ideal)} = I_j e^{l_i}$$

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Sinogram restoration for focal spot

- One could conceivably try to deconvolve the focal spot effect entirely.
  - This kind of superresolution approach is likely to be highly ill-posed.
- A more reasonable strategy is to attempt to *equalize* resolution across the sinogram by compensating for the differential focal spot effects.
  - To do this, we discretize the scatter plots by smoothing and subsampling to obtain coefficients linking the target, uniform-resolution sinogram to the measured, non-uniform resolution sinogram.
- I..e. we seek coefficients b_{ij} such that we can write:

$$I_i^{(meas)} = \sum_j^{N_y} b_{ij} I_j^{(unif)}$$

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An imaging model

- We assume the CT scan produces a set of measurements that are realizations of random variables:

Object attenuation map

Scatter

Dark current

Gain

Energy bin

$$Y_i^{meas} \cong G_i \sum_{m=1}^{M} E_m \text{Poisson}\left\{ \sum_{j=1}^{N_y} b_{ij} I_j \lambda_m^{(j)} \exp\left[-\int_{L_j} \mu(\mathbf{x}, E_m)\, dl \right] + s_m^{(i)} \right\} + \text{Normal}\{ d_i, \sigma_i^2 \}$$

Spectrum

Electronic noise

Degradation kernel

Incident beam intensity

Attenuation Line integral

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A simplified imaging model

- More practically, we assume the CT scan produces a set of measurements that are realizations of random variables:

Beam hardening function

$$Y_i^{meas} \simeq G_i \overline{E}_i \text{Poisson}\left\{ \sum_{j=1}^{N_y} b_{ij} I_j e^{-f_j(l_j)} + \overline{s}_i \right\} + \text{Normal}\{ d_i, \sigma_i^2 \}$$

$$\overline{E}_i = \sum_{m=1}^{M} \mathrm{E}_m \lambda_m^{(i)}$$

$$l_i \equiv \int_{L_i} \mu(\mathbf{r}, \overline{E}_i) dl = A\mathbf{x}$$

$$\overline{s}_i = \frac{1}{\overline{E}_i} \sum_{m=1}^{M} \mathrm{E}_m s_m^{(i)}$$

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Four potential strategies

1. Current commercial approach: Attempt to estimate the line integrals from the data by standard sinogram preprocessing/calibration techniques and then use analytic reconstruction to obtain the image vector **x**.
2. Promising iterative approach: Attempt to estimate the line integrals from the data by standard sinogram preprocessing/calibration techniques and then use iterative reconstruction with statistical modeling to obtain the image vector **x**.
3. Pipe dream iterative approach: Use iterative reconstruction to estimate the image vector **x** directly from the transmission measurements by modeling all effects.
4. Our approach: Use iterative methods with statistical modeling to estimate the line integrals and then use analytic reconstruction to obtain the image vector **x**.

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Our approach to sinogram processing

- We have formulated CT sinogram preprocessing as a statistical restoration problem.
  - The goal is to estimate as accurately as possible the attenuation line integrals needed for reconstruction from the set of noisy, degraded measurements.
  - We do so by maximizing a penalized-likelihood objective function.
  - Reconstruction is then done by use of existing methods.
- The hope is that one could achieve reduced noise and artifact levels relative to existing approaches, especially in low-dose and non-contrast scans.

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Imaging model

- We assume the CT scan produces a set of measurements that are realizations of random variables:

Beam hardening function

Scatter

Dark current

Gain

$$Y_i^{meas} \cong G_i \times \text{Poisson}\left\{\sum_{j=1}^{N_y} I_j b_{ij} e^{-f(l_j)} + s_i\right\} + \text{Normal}\left\{d_i, \sigma_i^2\right\}$$

Incident beam intensity

Degradation kernel

Attenuation Line integral

Electronic noise

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Adjusted measurements

- The sum of a Poisson and a Gaussian does not produce a tractable likelihood function, so we define adjusted measurements that we assume to be Poisson:

$$Y_i \equiv \left[\left(\frac{Y_i^{meas} - d_i}{G_i}\right) + \frac{\sigma_i^2}{G_i^2}\right]$$

$$\cong \text{Poisson}\left\{\sum_{j=1}^{N_y} I_j b_{ij} e^{-f(l_j)} + s_i + \frac{\sigma_i^2}{G_i^2}\right\}$$

- A similar approximation is employed by Snyder *et al.* in the context of CCD image restoration. (JOSA 1993)

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Objective function

- We find the undegraded attenuation line integrals by

$$\hat{\mathbf{l}} = \underset{\mathbf{l} \geq \mathbf{0}}{\arg\max} \left[L(\mathbf{l};\mathbf{y}) - \beta R(\mathbf{l}) \right]$$

- Here $L(\mathbf{l};\mathbf{y})$ is the Poisson likelihood for the adjusted measurements $\mathbf{y}$ and $R(\mathbf{l})$ is the roughness penalty.
- To maximize we make use of an update derived by use of the optimization transfer approach (Fessler, 2000) adapting some tricks due to DePierro (1995).

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Penalized likelihood sinogram restoration

- Perform a sinogram-domain penalized likelihood estimate of ideal line integrals needed for reconstruction.

Gain

Beam hardening function

Scatter

Dark current

$$Y_i^{meas} \cong G_i \times \mathrm{Poisson}\left\{\sum_{j=1}^{\mathrm{N_y}} I_j b_{ij} e^{-f(l_j)} + s_i\right\} + \mathrm{Normal}\left\{d_i, \sigma_i^2\right\}$$

Incident beam intensity

Degradation kernel

Attenuation Line integral

Electronic noise

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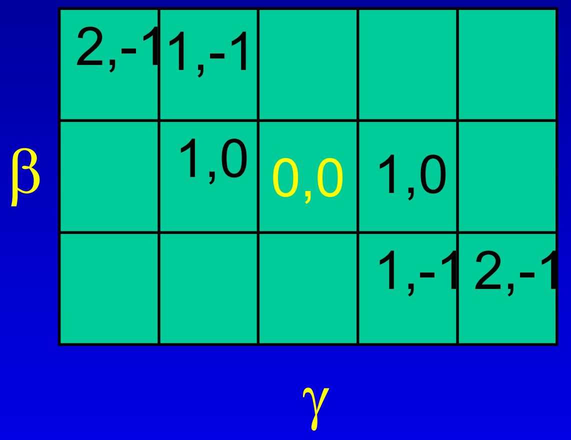
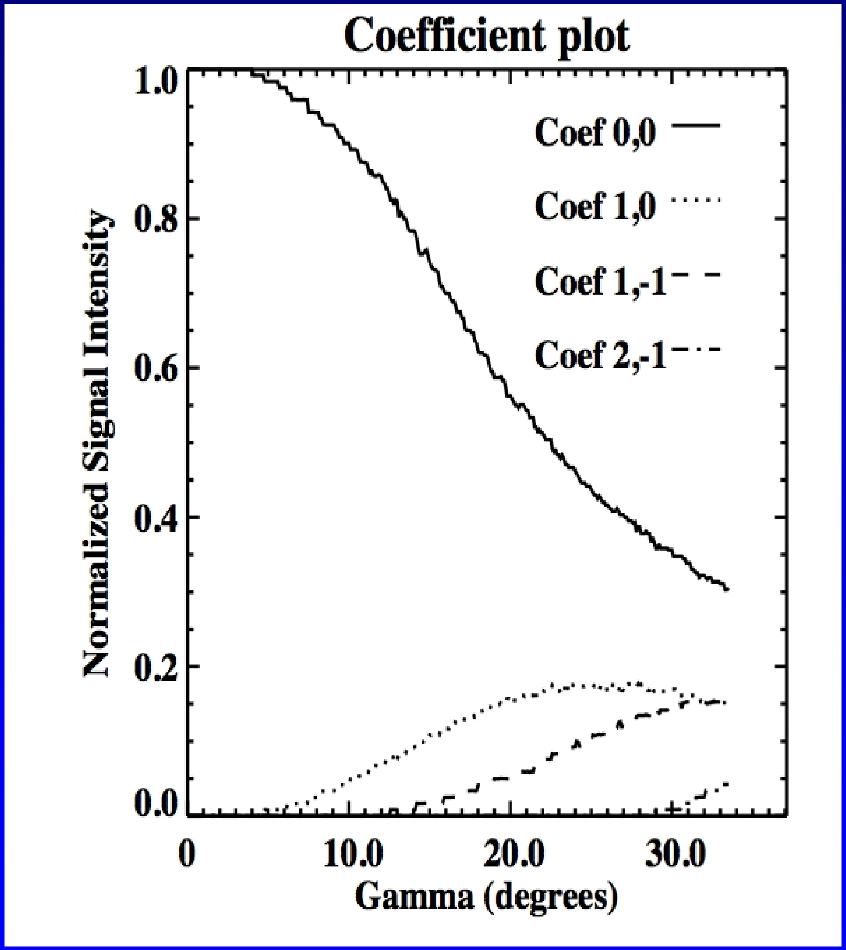
Plot of coefficients versus angle in fanbeam





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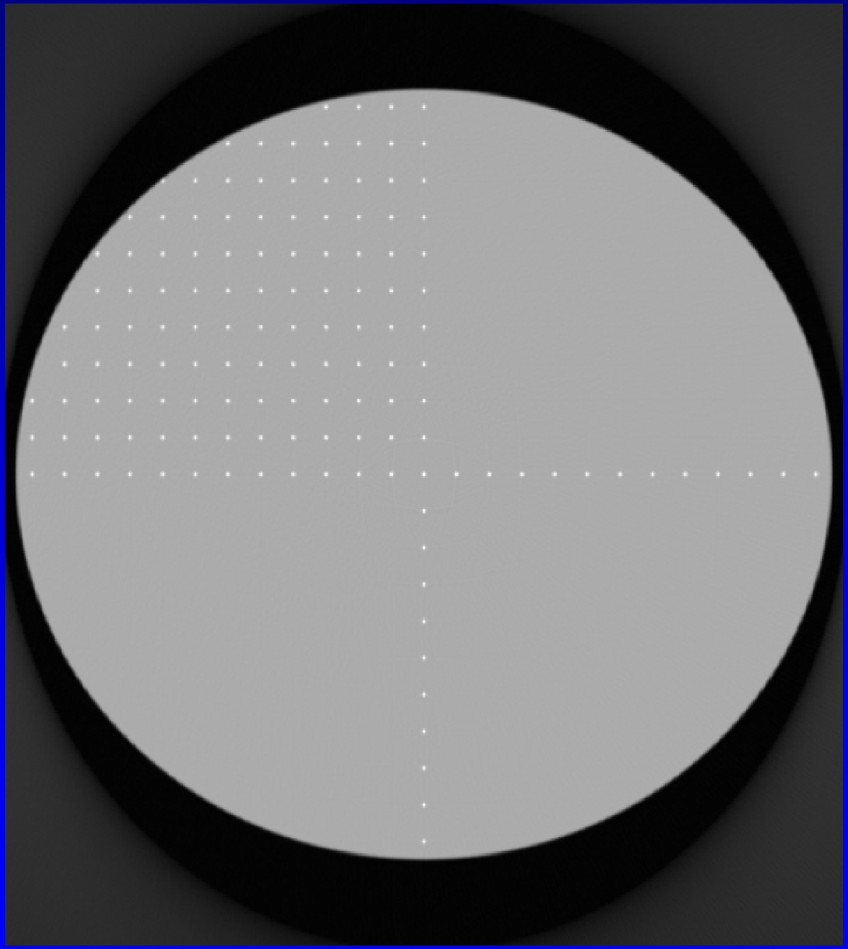
These are the coefficients we find, as a function of fanbeam coordinate.

Impulse phantom

- We simulated projections of this phantom in a variety of ways using Radonis package from Philips R&D.
- We then calculated the FWHM and FWTM of the impulses.
- We plot these versus radial distance.



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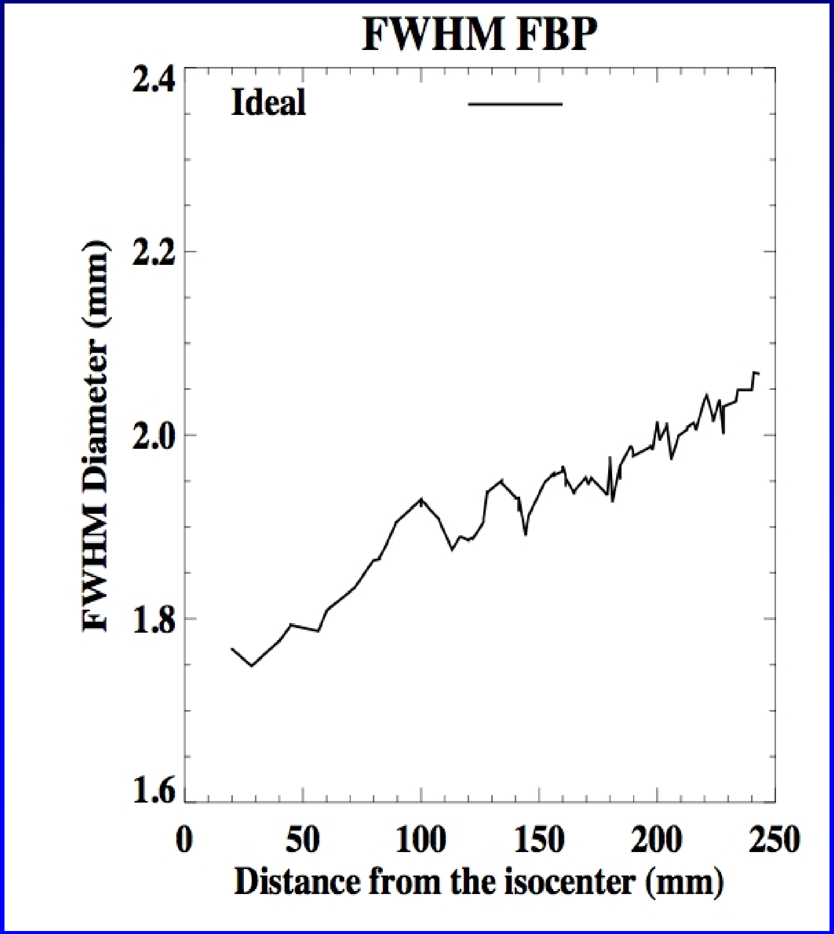
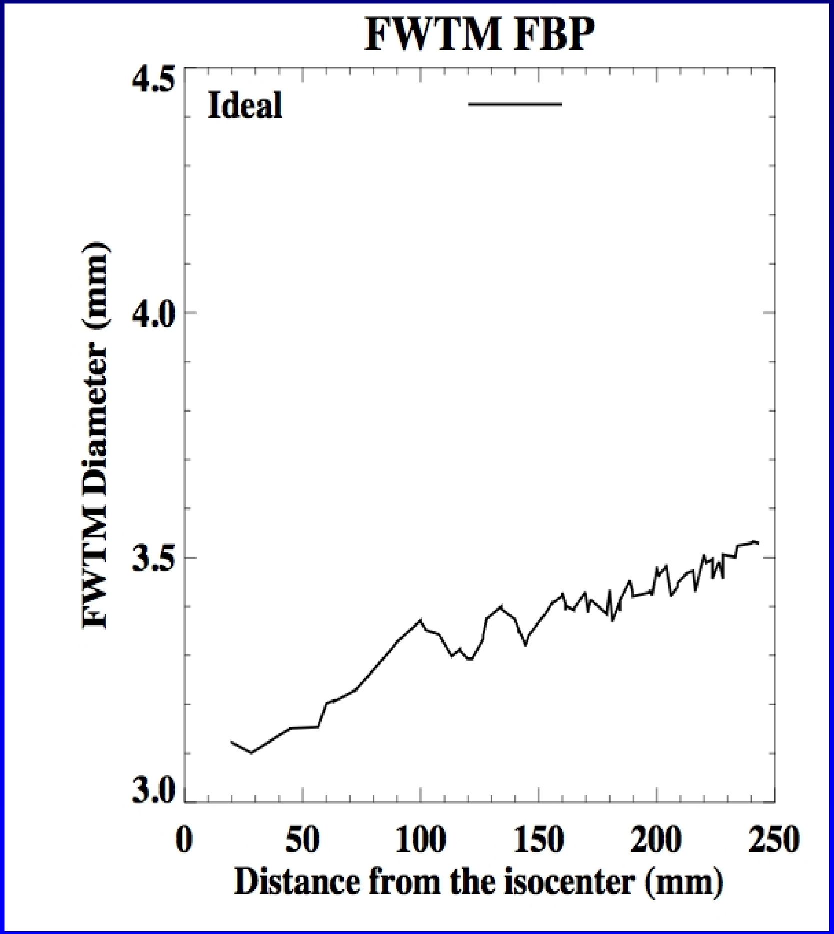
Impulse properties versus distance from isocenter





Ideal data is simulated with no focal spot angulation (i.e., 90 degrees).

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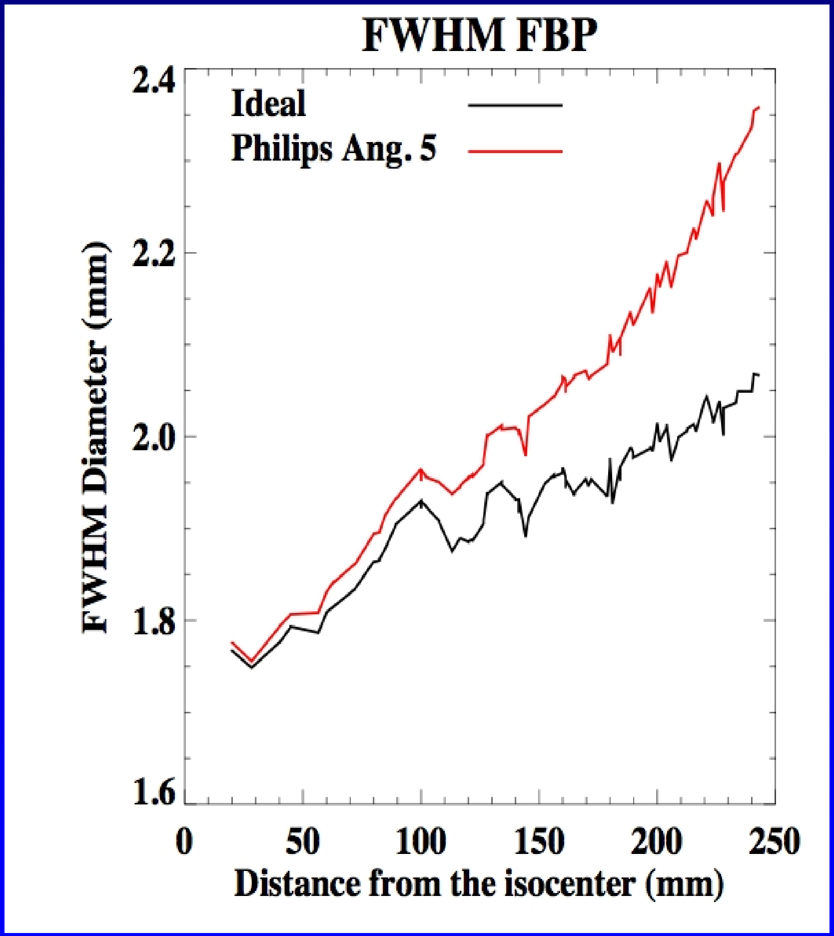
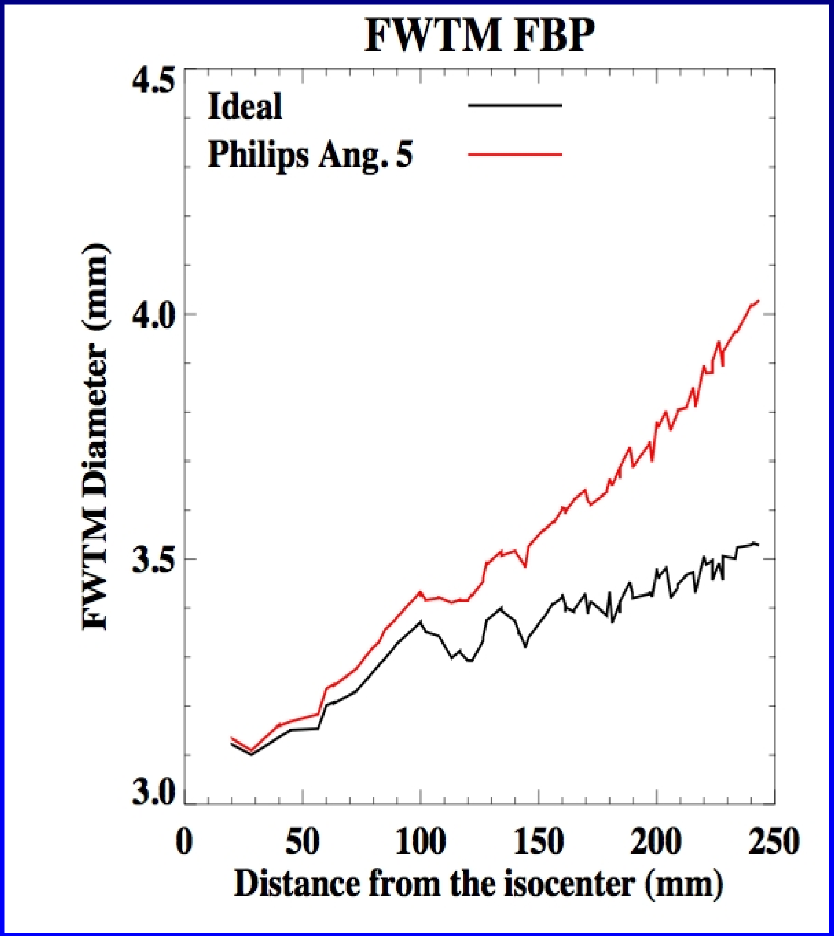
Impulse properties versus distance from isocenter





Re... ents data simulated with 5 degree anode angle.

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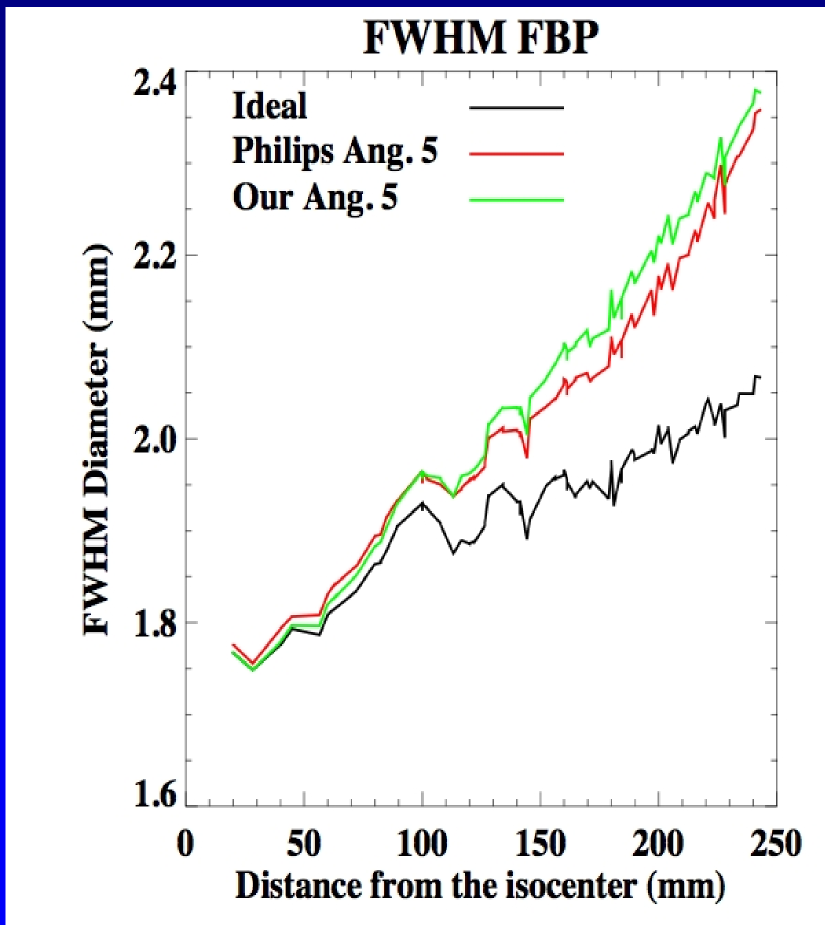
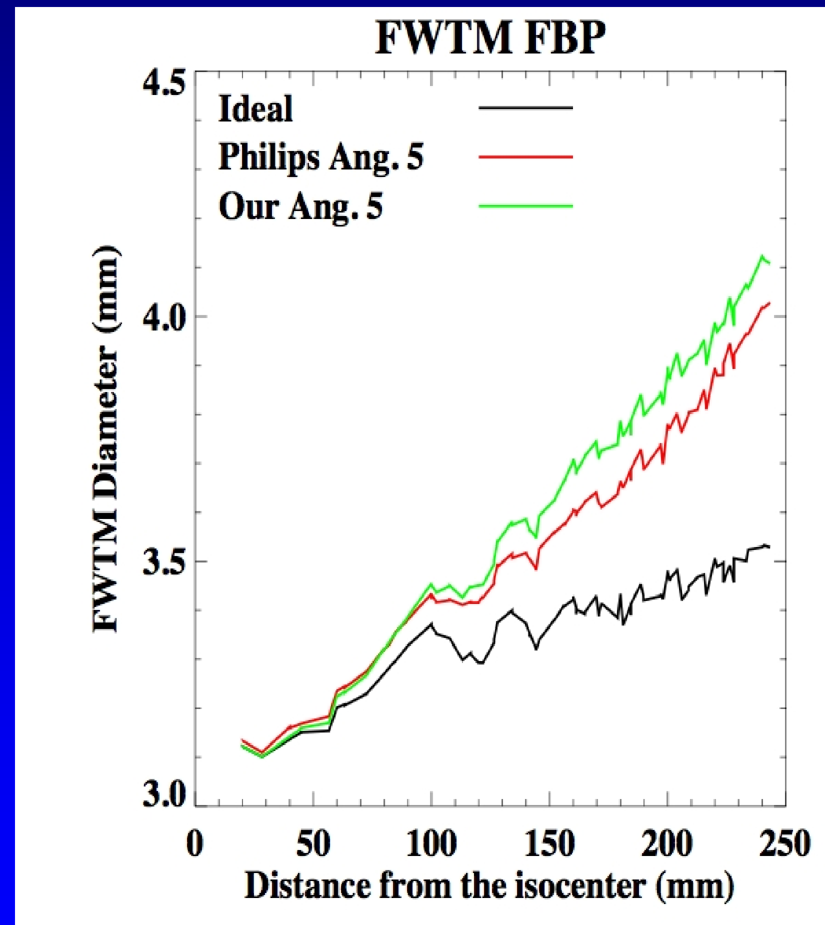
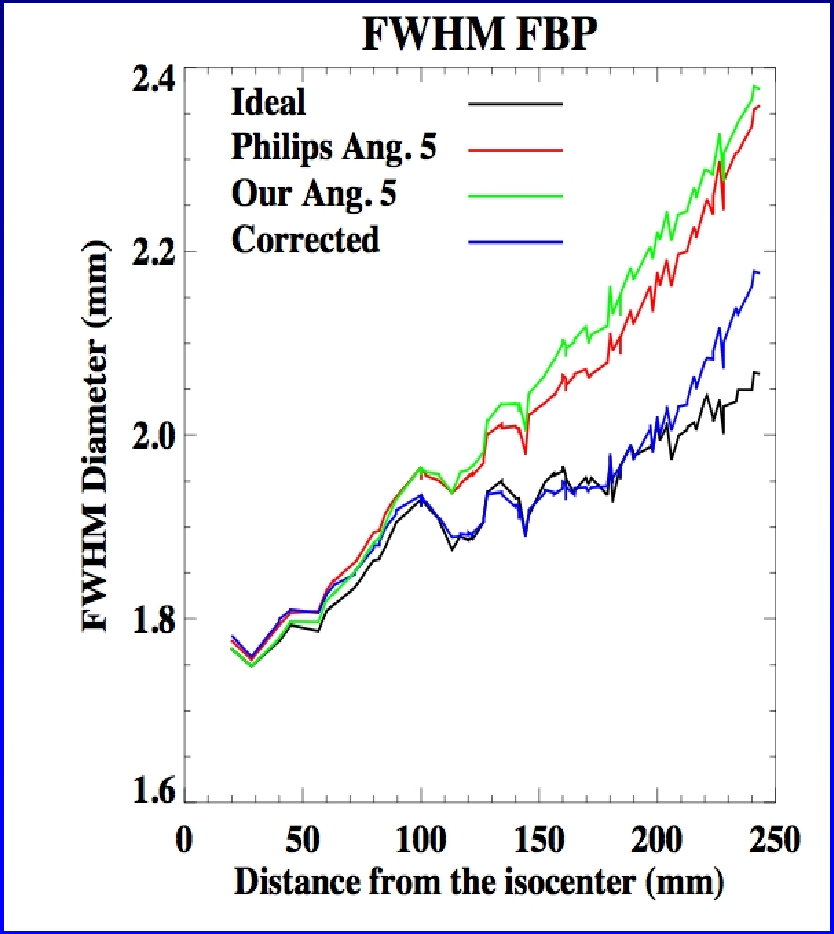
Impulse properties versus distance from isocenter

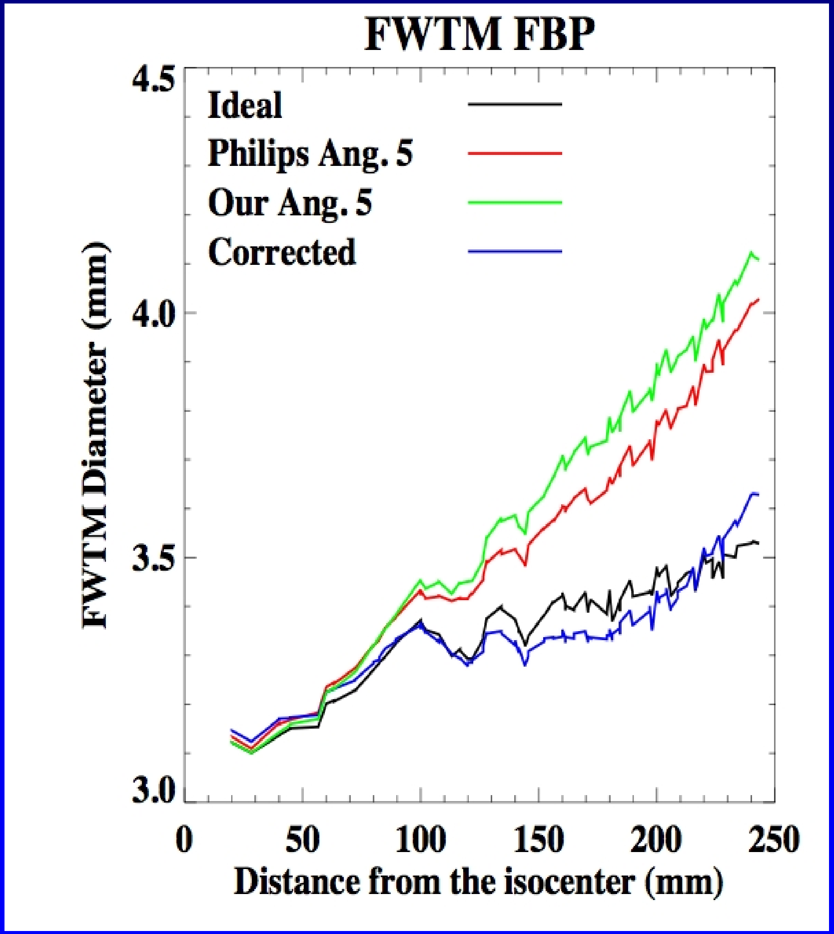




Green data obtained after applying our coefficients to ideal data.

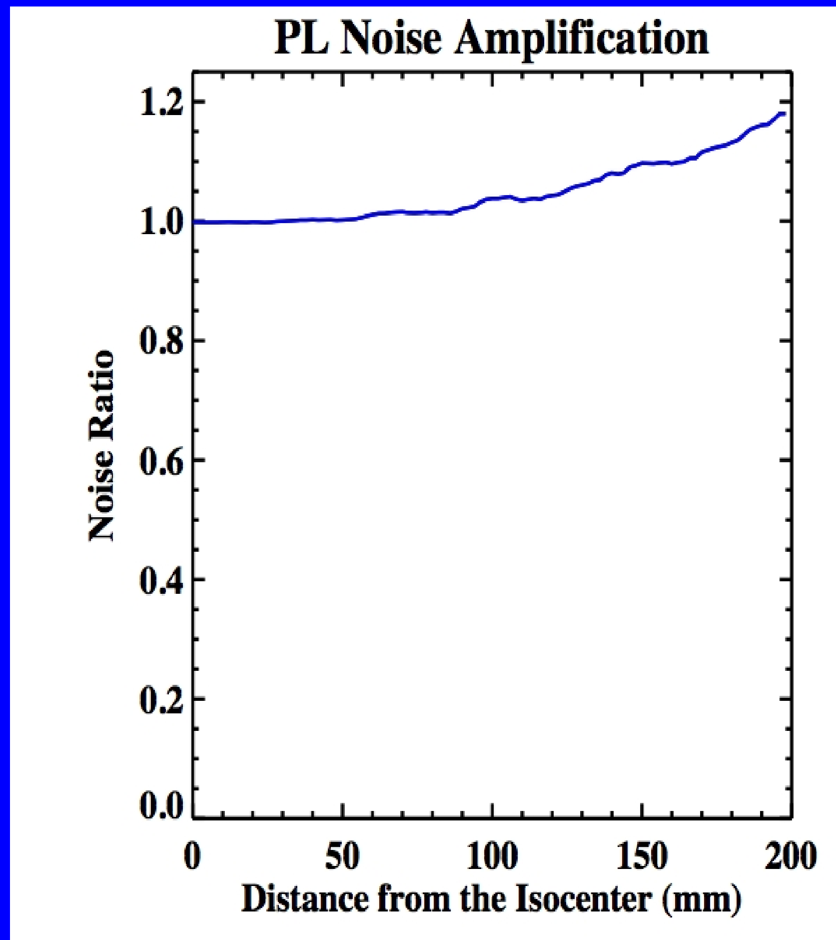
Impulse properties versus distance from isocenter





Blue [illegible]a obtained after applying our correction to red curve data.

In the presence of noise



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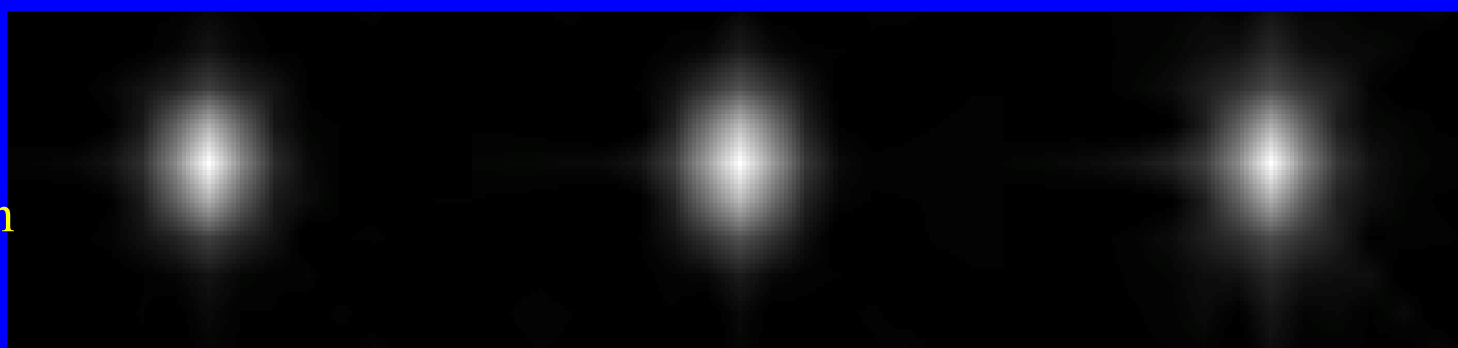
Comparison to fully iterative methods

Typical noisy images

Image domain | Sinogram domain | Approx. sino domain

Local impulse response in hot circle



We used closed-form expressions for the PWLS methods to calculate noisy images, local impulse response functions, and variance maps for the three approaches for a 128^2X128^2 matrix P. We used strip integrals to calculate the forward-projected data and FBP in place of an explicit P^{-1}.

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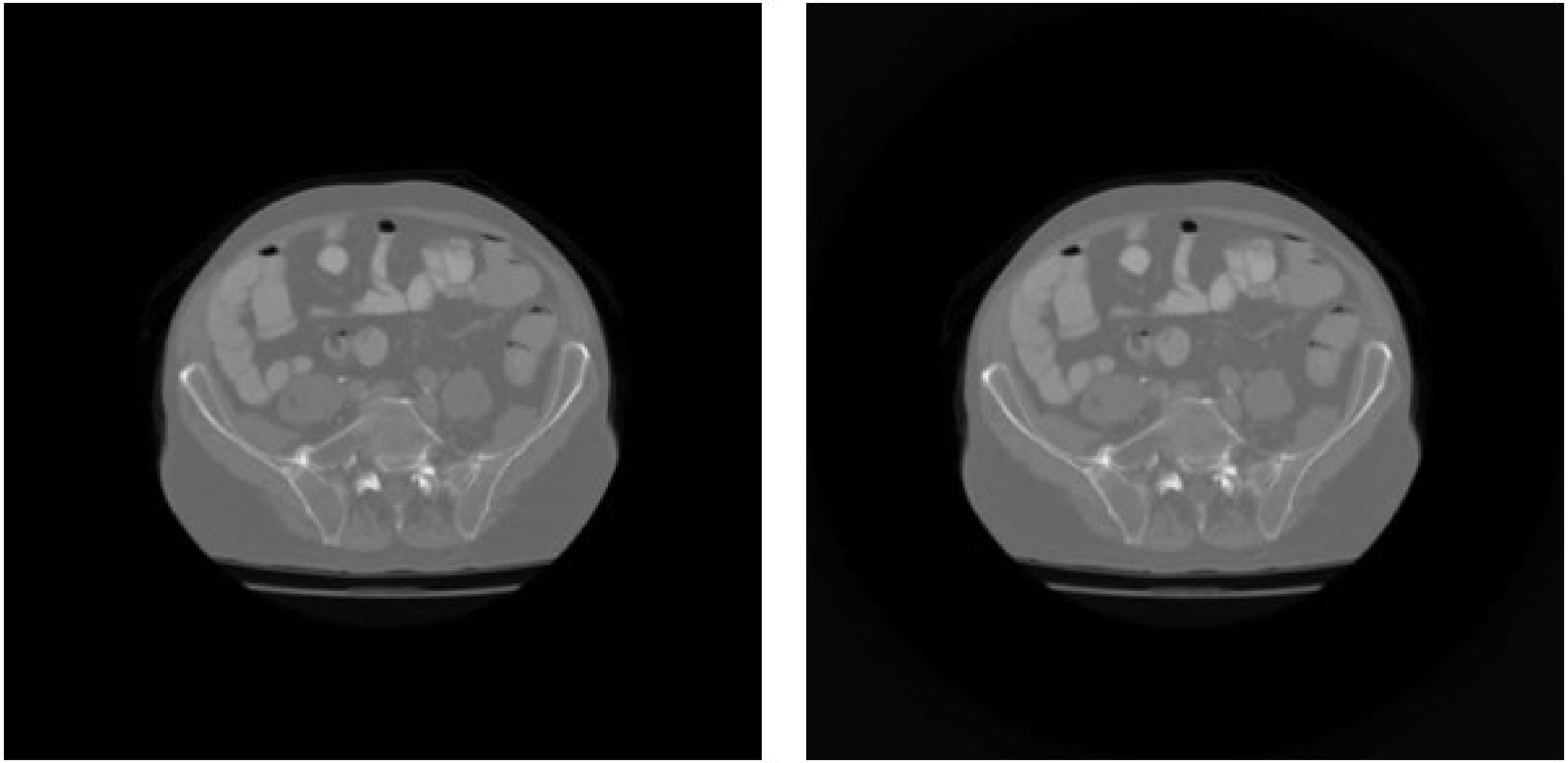
Comparison to fully iterative methods



(c) Image-domain

(d) Sinogram-domain

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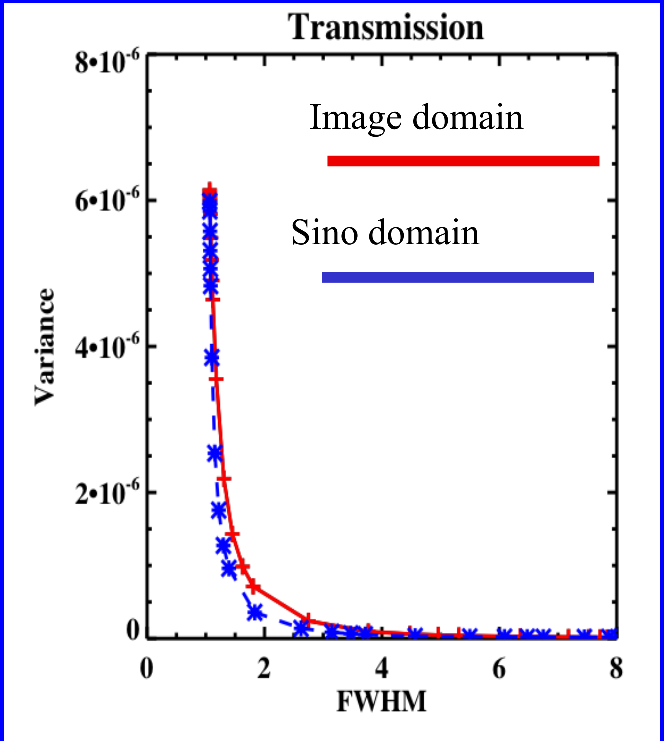
Comparison to fully iterative methods



- Through careful “penalty matching” resolution-variance performance can be very similar.
- Computation is about 100 times faster for sinogram domain.

Limitations:

- Can’t easily enforce image-domain non-negativity in sinogram space.
- Can’t easily enforce image-domain edge-preserving priors in sinogram space.

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Conclusions

- **Sinogram restoration is a statistically principled way of preprocessing CT data prior to reconstruction by analytic algorithms**
- **It can not only be used to reduce artifacts (the point of most preprocessing) but also to reduce noise and model geometric effects, as in fully iterative algorithms.**
- **At the very least, it represents a computationally efficient middle ground between analytic algorithms and fully iterative ones.**
  - **However, in many studies we have done it performs as well as fully iterative algorithms.**
  - **It can readily accommodate the proper "nonlinear" model for CT. Many/Most fully iterative algorithms work with linearized system models.**
- **Naturally it has limitations: harder to accommodate non-negativity and other constraints, harder to implement edge-preserving priors.**

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