Metrics for Evaluation of Segmentation Algorithms

Karina Bond, Jeff Kallman, Steve Azevedo, Harry E. Martz, Jr.
Lawrence Livermore National Laboratory
LLNL-PRES-557172-DRAFT
(IM#610872)

April 22, 2012
Version 11
• One way to measure reconstruction performance is to measure how well the result can be segmented.

• We have been studying how to measure segmentation performance. It turns out this is not a trivial task.

• We surveyed the published literature on segmentation evaluation metrics and have developed a few ideas of our own.

• We describe one of the segmentation evaluation metrics we developed.

• We present the results of applying this metric to the Segmentation Initiative.
Evaluation of Segmentation Algorithms

**Segmentation Evaluation Methods**

- **Subjective Methods**
  Qualitative evaluation of segmentation results by a human evaluator.

- **Objective Methods**
  Quantitative evaluation of segmentation algorithms.

- **System-Level Methods**
  Methods that evaluate segmentation on the basis of the larger system's parameters. In the case of CT based images these parameters might be the following:
  - $\mu(i)$, Linear Attenuation Coefficient of $i$-th object.
  - $V(i)$, volume of $i$-th object.

- **Direct Methods**
  Methods that evaluate segmentation independent of the larger system they are used in.

- **Analytical Methods**
  Theoretical evaluation methods that can be calculated without any results solely based on algorithm details.

- **Empirical Methods**
  Evaluation Methods that are calculated on the basis of the results of the segmentation algorithm.

- **Unsupervised Methods**
  Evaluations methods that are based only on a set of segmentation results (no ground truth).

- **Supervised Methods**
  Evaluation methods that are based on the result of the segmentation algorithm and a ground truth image.
  - P1\P2 Metric
  - Martin Error (GCE\LCE)
  - Object Consistency Error (OCE)
  - F-Measure
Assume $I_g = \{T_1, T_2, \ldots, T_M\}$ is the ground truth image, where $T_i$ is the i-th object in $I_g$.

Assume $I_s = \{S_1, S_2, \ldots, S_N\}$ is the segmented image, where $S_j$ is the j-th segment in $I_s$.

Precision, $P_{ij}$ and Recall, $R_{ij}$ for the $ij$-th fragment, $G_{ij}$ can be calculated as follows.

For $1 \leq i \leq M$ and $1 \leq j \leq N$

\[
G_{ij} = T_i \cap S_j
\]

\[
R_{ij} = \frac{|G_{ij}|}{|T_i|} = \frac{|T_i \cap S_j|}{|T_i|}
\]

\[
P_{ij} = \frac{|G_{ij}|}{|S_j|} = \frac{|T_i \cap S_j|}{|S_j|}
\]
Assume $I_g = \{T_1, T_2, \ldots, T_M\}$ is the ground truth image, where $T_i$ is the i-th object in $I_g$.

Assume $I_s = \{S_1, S_2, \ldots, S_N\}$ is the segmented image, where $S_j$ is the j-th segment in $I_s$.

Precision, $P_{ij}$ and Recall, $R_{ij}$ for the $ij$-th fragment, $G_{ij}$ can be calculated as follows.

For $1 \leq i \leq M$ and $1 \leq j \leq N$

$$G_{ij} = T_i \cap S_j$$

$$R_{ij} = \frac{|G_{ij}|}{|T_i|} = \frac{|T_i \cap S_j|}{|T_i|}$$

$$P_{ij} = \frac{|G_{ij}|}{|S_j|} = \frac{|T_i \cap S_j|}{|S_j|}$$
Assume $I_g = \{T_1, T_2, \ldots, T_M\}$ is the ground truth image, where $T_i$ is the $i$-th object in $I_g$.

Assume $I_s = \{S_1, S_2, \ldots, S_N\}$ is the segmented image, where $S_j$ is the $j$-th segment in $I_s$.

Precision, $P_{ij}$ and Recall, $R_{ij}$ for the $ij$-th fragment, $G_{ij}$ can be calculated as follows.

For $1 \leq i \leq M$ and $1 \leq j \leq N$

$$G_{ij} = T_i \cap S_j$$

$$R_{ij} = \frac{|G_{ij}|}{|T_i|} = \frac{|T_i \cap S_j|}{|T_i|}$$

$$P_{ij} = \frac{|G_{ij}|}{|S_j|} = \frac{|T_i \cap S_j|}{|S_j|}$$

<table>
<thead>
<tr>
<th>R</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>0.75</td>
<td>0.25</td>
<td>0</td>
</tr>
<tr>
<td>T2</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>T3</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>P</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>1</td>
<td>0.2</td>
<td>0</td>
</tr>
<tr>
<td>T2</td>
<td>0</td>
<td>0.8</td>
<td>0</td>
</tr>
<tr>
<td>T3</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>
The F-Measure [1] is calculated for each fragment from their precision and recall as follows,

\[
F_{ij} = \frac{2P_{ij}R_{ij}}{(P_{ij} + R_{ij})}
\]

when \( P_{ij} \neq 0, R_{ij} \neq 0 \)

\[ F_{ij} = 0 \]

Otherwise.

In order to get one quantitative metric per dataset, we calculate a combined F-Measure as

\[
F_g = \frac{1}{M} \sum_{i=1}^{M} \sum_{j} \max(F_{ij})|T_i|
\]

\[
\text{Table:}
\begin{array}{c|ccc}
  & S1 & S2 & S3 \\
  \hline
  \text{T1} & 0.75 & 0.25 & 0 \\
  \text{T2} & 0 & 1 & 0 \\
  \text{T3} & 0 & 0 & 1 \\
\end{array}
\]

\[
\begin{array}{c|ccc}
  & S1 & S2 & S3 \\
  \hline
  \text{T1} & 1 & 0.2 & 0 \\
  \text{T2} & 0 & 0.8 & 0 \\
  \text{T3} & 0 & 0 & 1 \\
\end{array}
\]

\[
F_g = 0.86*0.4 + 0.89*0.4 + 1*0.2
\]

\[
F_g = 0.9
\]

Training Bag 3
Precision vs. Recall

- Researcher 1
  - Precision vs. Recall

- Researcher 2
  - Precision vs. Recall

- Researcher 3
  - Precision vs. Recall

- Researcher 4
  - Precision vs. Recall

- Researcher 5
  - Precision vs. Recall

- Back Ground
- toothpaste tube
- Mens Sneaker - R
- Mens Sneaker - L
- Flat Iron
- CD's
- Bar Soap
- Candles - tealight
- Toothbrushes - 4 pk
- Leather Jacket
- Rubber (harder)
- Magazine - GH
- Skip Bo
Researchers’ Scores for Training Bag 3

Fg for Training Bag 3

- Researcher 1
- Researcher 2
- Researcher 3
- Researcher 4
- Researcher 5
Summary of Scores

• Based on the Fg metric, all researcher scores are in the same ball park. There is no one researcher that outshines the others in performance.

• Since we have not been able tie these scores back to system – level performance, we cannot say that small differences in Fg scores make an insignificant difference to overall system performance.

• Researchers 1, 2 & 3 have a similar trend across all the bags. Researchers 4 & 5 have much more variation in their scores across all the bags. This means that the performance of Researcher’s 3 & 4 algorithms is not as consistent for varying data as Researcher’s 1, 2 & 3.
Applicability to System-level Performance

Segmentation Evaluation Methods

Subjective Methods
Qualitative evaluation of segmentation results by a human evaluator.

Objective Methods
Quantitative evaluation of segmentation algorithms.

System-Level Methods
Methods that evaluate segmentation on the basis of the larger system's parameters. In the case of CT based images these parameters might be the following.

- \( \mu(i) \), Linear Attenuation Coefficient of the \( i \)-th object.
- \( V(i) \), Volume of the \( i \)-th object.

Direct Methods
Methods that evaluate segmentation independent of the larger system they are used in.

Analytical Methods
Theoretical evaluation methods that can be calculated without any results solely based on algorithm details.

Empirical Methods
Evaluation methods that are calculated on the basis of the results of the segmentation algorithm.

Unsupervised Methods
Evaluation methods that are based only on a set of segmentation results (no ground truth).

Supervised Methods
Evaluation methods that are based on the result of the segmentation algorithm and a ground truth image.

- \( P1\|P2 \) Metric
- Martin Error (GCE\|LCE)
- Object Consistency Error (OCE)
- F-Measure

It is important that supervised metrics correlate well with system performance.
• For CT and ATD, we really need to identify threats based on system-level values per segment
  • linear attenuation coefficient ($\mu$) and
  • volume ($V$) of the segment

• As segmentation gets worse a good metric should also get worse.
  • Over-segmenting (splitting) can lead to correct $\mu$ and wrong $V$, while
  • Under-segmenting (merging) can lead to wrong $\mu$ and wrong $V$

• Current metric definitions allow a segment to match with more than one ground-truth object
  • Errors are calculated per ground-truth object (not per segment)
  • As the red segment merges more into Ground Truth Object 1, segmentation get worse but the current metrics get better after initially getting worse.

We will need to modify these supervised metrics to make them more appropriate for system-level and ATD performance.
• Summary
  • One way to measure reconstruction performance is to measure how well the result can be segmented.
  • It turns out that measuring segmentation performance is not a trivial task.
  • Surveyed the published literature on segmentation evaluation metrics and have developed a few ideas of our own.
  • Described one of the segmentation evaluation metrics we developed.
  • Presented the results of applying this metric to the Segmentation Initiative researchers' results

• Future work
  • Develop a segmentation metric that can be related back to system-level parameters.
Backup Slides
Assume $I_g = \{T_1, T_2, \ldots, T_M\}$ is the ground truth image, where $T_i$ is the i-th object in $I_g$.

Assume $I_s = \{S_1, S_2, \ldots, S_N\}$ is the segmented image, where $S_j$ is the j-th segment in $I_s$.

Precision, $P_{ij}$ and Recall, $R_{ij}$ for the $ij$-th fragment, $G_{ij}$ can be calculated as follows.

For $1 \leq i \leq M$ and $1 \leq j \leq N$

$$G_{ij} = T_i \cap S_j$$

$$R_{ij} = \frac{|G_{ij}|}{|T_i|} = \frac{|T_i \cap S_j|}{|T_i|}$$

$$P_{ij} = \frac{|G_{ij}|}{|S_j|} = \frac{|T_i \cap S_j|}{|S_j|}$$

<table>
<thead>
<tr>
<th>R</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>T2</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>T3</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>P</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>T2</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>T3</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>
Assume $I_g = \{T_1, T_2, \ldots, T_M\}$ is the ground truth image, where $T_i$ is the i-th object in $I_g$.

Assume $I_s = \{S_1, S_2, \ldots, S_N\}$ is the segmented image, where $S_j$ is the j-th segment in $I_s$.

Precision, $P_{ij}$ and Recall, $R_{ij}$ for the $ij$-th fragment, $G_{ij}$ can be calculated as follows.

For $1 \leq i \leq M$ and $1 \leq j \leq N$

\[
G_{ij} = T_i \cap S_j
\]

\[
R_{ij} = \frac{|G_{ij}|}{|T_i|} = \frac{|T_i \cap S_j|}{|T_i|}
\]

\[
P_{ij} = \frac{|G_{ij}|}{|S_j|} = \frac{|T_i \cap S_j|}{|S_j|}
\]

Precision

<table>
<thead>
<tr>
<th>R</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>T2</td>
<td>0</td>
<td>0.5</td>
<td>0.5</td>
<td>0</td>
</tr>
<tr>
<td>T3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Recall

<table>
<thead>
<tr>
<th>R</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>T2</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>T3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>
Assume \( I_g = \{T_1, T_2, \ldots, T_M\} \) is the ground truth image, where \( T_i \) is the i-th object in \( I_g \).

Assume \( I_s = \{S_1, S_2, \ldots, S_N\} \) is the segmented image, where \( S_j \) is the j-th segment in \( I_s \).

Precision, \( P_{ij} \) and Recall, \( R_{ij} \) for the \( ij \)-th fragment, \( G_{ij} \) can be calculated as follows.

For \( 1 \leq i \leq M \) and \( 1 \leq j \leq N \)

\[
G_{ij} = T_i \cap S_j
\]

\[
R_{ij} = \frac{|G_{ij}|}{|T_i|} = \frac{|T_i \cap S_j|}{|T_i|}
\]

\[
P_{ij} = \frac{|G_{ij}|}{|S_j|} = \frac{|T_i \cap S_j|}{|S_j|}
\]
The P1 and P2 Metrics are calculated for each fragment from their precision and recall as follows,

\[ p_{1ij} = P_{ij} R_{ij} \]

\[ p_{2ij} = P_{ij} R_{ij}^2 \]

In order to get one quantitative metric per dataset, we calculate a combined P1 and P2 score as,

\[ P1 = \frac{1}{M} \sum_{i=1}^{M} \sum_{j=1}^{N} P_{ij} R_{ij} \]

\[ P2 = \frac{1}{M} \sum_{i=1}^{M} \sum_{j=1}^{N} P_{ij} R_{ij}^2 \]

\[
\begin{array}{c|ccc}
R & S1 & S2 & S3 \\
\hline
T1 & 0.75 & 0.25 & 0 \\
T2 & 0 & 1 & 0 \\
T3 & 0 & 0 & 1 \\
\end{array}
\]

\[
\begin{array}{c|ccc}
P & S1 & S2 & S3 \\
\hline
P1 & 1 & 0.2 & 0 \\
P2 & 0.75 & 0.05 & 0 \\
\end{array}
\]

\[
\begin{array}{c|ccc}
P1 & S1 & S2 & S3 \\
\hline
P1 & 0.75 & 0.05 & 0 \\
P2 & 0.56 & 0.01 & 0 \\
\end{array}
\]

\[
\begin{array}{c|ccc}
P1 & S1 & S2 & S3 \\
\hline
P1 & 0.8667 \\
P2 & 0.7917 \\
\end{array}
\]
Metric based on Object-level Consistency Error (OCE)

Using previous definitions of $I_g, I_s, M, N, T_i, S_j$

$$E_{gs}(I_g, I_s) = E_{gs} = \sum_{i=1}^{M} \left[ 1 - \sum_{j=1}^{N} \frac{\left| T_i \cap S_j \right|}{\left| T_i \cup S_j \right|} \times W_{ij} \right] W_i$$

$$W_{ij} = \frac{\bar{\delta}\left(\left| T_i \cap S_j \right|\right)}{\sum_{k=1}^{N} \bar{\delta}\left(\left| T_i \cap S_k \right|\right)}$$

$$W_i = \frac{|T_i|}{\sum_{l=1}^{M} |T_l|}$$

$$OCE = \min\left(E_{gs}, E_{sg}\right)$$

$$E'_{gs} = 1 - E_{gs}$$

where, $\bar{\delta}(x) = 1 - \delta(x)$ and $\delta(x)$ is the delta function whose value equals 1 if the input is 0, and whose value is 1 otherwise.

Segmentation 1 and Segmentation 2 are very similar segmentations of the same input image, so their scores should be similar. P1, P2, Fg produce similar scores for similar segmentations. E’gs produces dissimilar scores for similar segmentations. Therefore, E’gs is not a useful metric for the evaluation of segmentation.
Test case 1 and Test Case 2 are symmetric – ground truth for Test 1 is the segmentation for Test 2 and segmentation for Test 1 is the ground truth for Test 2.

- P1 and P2 give different scores for Test 1 and Test 2.
- E’gs and Fg give the same scores for Test 1 and Test 2.
Precision vs. Recall, Researcher 1, Training bag 3

Precision vs. Recall

- Back Ground
- Toothpaste tube
- Mens Sneaker - R
- Mens Sneaker - L
- Flat Iron
- CD's
- Bar Soap
- Candles - tealight
- Toothbrushes - 4 pk
- Leather Jacket
- Rubber (harder)
- Magazine - GH
- Skip Bo
Precision vs. Recall, Researcher 2, Training bag 3

Graph showing precision vs. recall for various items:
- Back Ground
- toothpaste tube
- Mens Sneaker - R
- Mens Sneaker - L
- Flat Iron
- CD's
- Bar Soap
- Candles - tealight
- Toothbrushes - 4 pk
- Leather Jacket
- Rubber (harder)
- Magazine - GH
- Skip Bo
Precision vs. Recall, Researcher 3, Training bag 3

**Graph:**

- **Axes:**
  - Y-axis: Precision
  - X-axis: Recall

- **Legend:**
  - Back Ground
  - Toothpaste tube
  - Mens Sneaker - R
  - Mens Sneaker - L
  - Flat Iron
  - CD's
  - Bar Soap
  - Candles - tealight
  - Toothbrushes - 4 pk
  - Leather Jacket
  - Rubber (harder)
  - Magazine - GH
  - Skip Bo
Precision vs. Recall, Researcher 4, Training bag 3

**Researcher 4**

**Precision vs. Recall**

- Back Ground
- Toothpaste tube
- Mens Sneaker - R
- Mens Sneaker - L
- Flat Iron
- CD's
- Bar Soap
- Candles - tealight
- Toothbrushes - 4 pk
- Leather Jacket
- Rubber (harder)
- Magazine - GH
- Skip Bo
Precision vs. Recall, Researcher 5, Training bag 3

The graph shows the precision vs. recall for various objects. The objects are categorized as follows:

- Back Ground
- Toothpaste tube
- Mens Sneaker - R
- Mens Sneaker - L
- Flat Iron
- CD's
- Bar Soap
- Candles - tealight
- Toothbrushes - 4 pk
- Leather Jacket
- Rubber (harder)
- Magazine - GH
- Skip Bo

The graph plots precision on the y-axis and recall on the x-axis, with different symbols representing each object category.
Applicability to System-level Metrics

As S2 bleeds into T1, error in volume and mean attenuation for S2 increases. Therefore we should expect that the scoring metric should decrease from left to right.
Applicability to System-level Metrics

- P2 and Fg decrease as the S2 bleeds into T1, until Precision and Recall for T1 are dominated by the Precision and Recall for the T1 vs. S2 fragment. After this point, P2 and Fg starts to increase even though intuitively the score should continue to decrease (since the segmentation continues to get worse).

This occurs because we are allowing the same segment (S2) to contribute to the score of more than one ground truth object (T1 and T2).
Proposed plan for developing a system-level applicable metric

Step 1: Assign each segment to a single ground truth object.

  - Hungarian algorithm to come up with the optimal assignment.

  - The cost can be based on the on multiple features such as overlap, distance between centroids, principal axes, distance to mean attenuation etc.

Step 2: Calculate a single metric by combining the individual “score” for each segment (w.r.t. to it’s assigned ground truth object from Step 1).

The individual score for each segment could be

  - It’s F-measure.
  - Mathew’s Correlation coefficient.
  - A multi-feature based error (i.e. error between the segment’s mean attenuation \(\text{volume}\) and it’s assigned ground truth object’s mean attenuation \(\text{volume}\)).