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Model-Based Ideas for Sensor Fusion

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Overview

- Formulating the fusion problem
- Parameterization
  - Results from Dual Energy CT study
- Model-based solution methods
  - Physics-based
  - Statistically-based

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Conclusions

- Model based methods have much to offer for fusion
- Principled approach to many issues associated with multi-sensor data acquisition, processing, and analysis
- Physics-based models allow for joint design and optimization of sensors and processing
- Statistically-based models allow for incorporation of prior information and exploitation of cross-modality correlations

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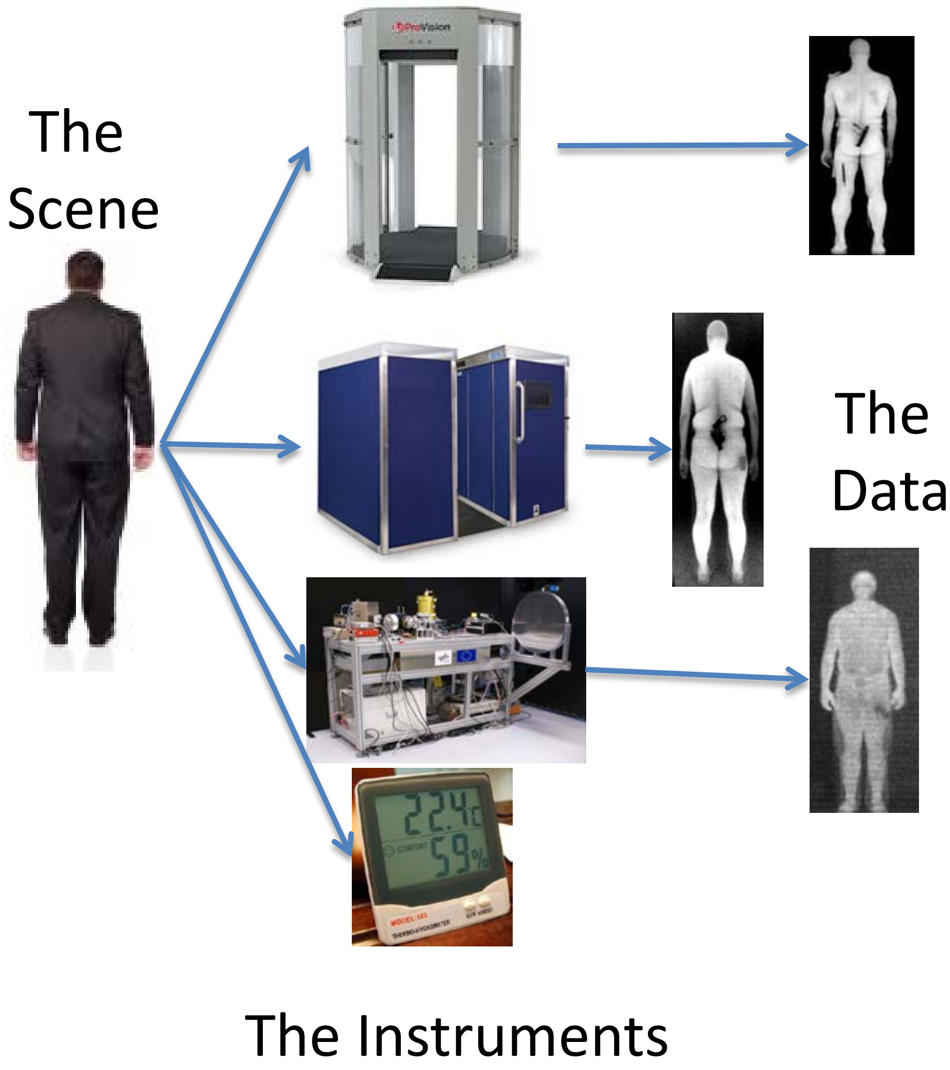
The Problem: Security



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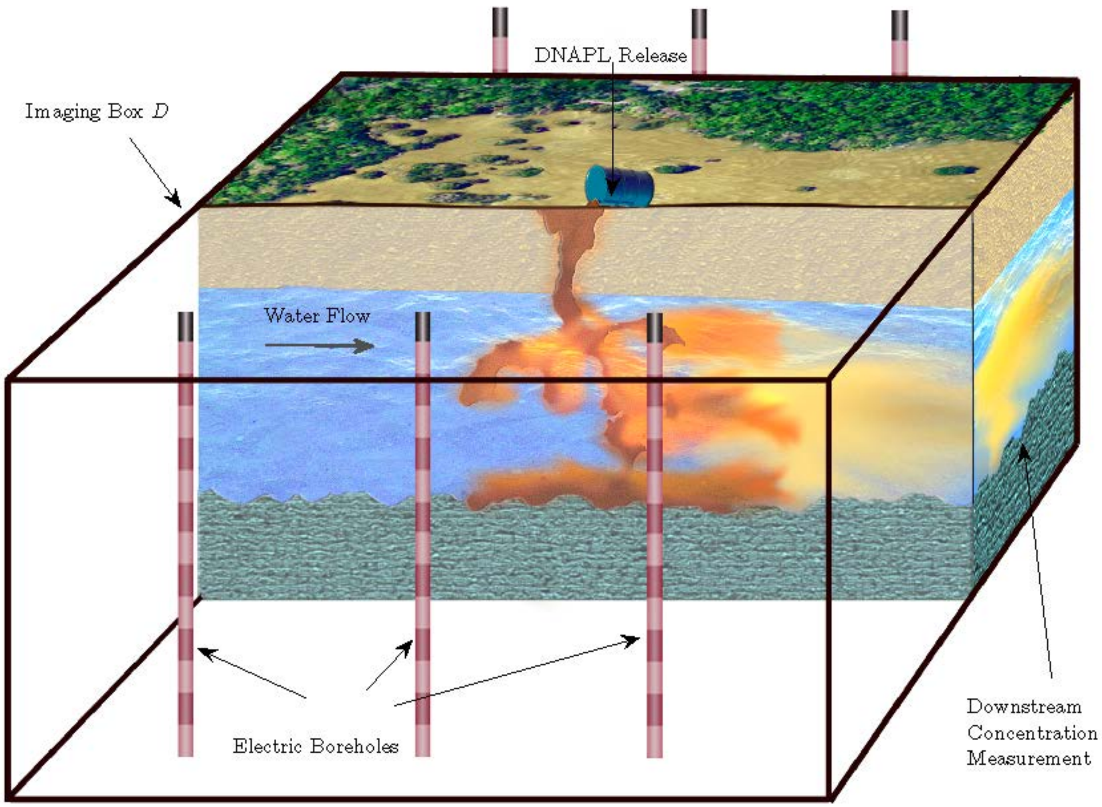
The Problem: Remediation



- The scene: subsurface distribution of contaminant saturation
- The instruments and data:
  - Downstream hydrogeochemical sampling of contaminant concentration
  - Cross-stream electrical resistance tomography

Common structure

- Everyone is looking at the same *scene*
- Each *instrument* produces data that is somehow related to a *property* of the scene
  - Security
    - "Reflectivity" or spectral structure for imaging-type modalities
    - Humidity, temperature, other environmental properties
    - Photoelectric and Compton scattering coefficients
  - Remediation
    - Electrical properties
    - Chemical composition
- Mapping from property to data can be highly complex, perhaps unknown, function of time, space, wavelength, etc.
- Goal: Recover some aspect about the scene in a manner that reflects the information in the various sources of data

What do we want?

- Important to be precise about what we want from the data
  - Presence of a material
  - Rough characterization (e.g., centroid and mass)
  - Detailed image of the scene
- Why?
  - Desired information should impact the design of the processing and perhaps even the instruments
  - May be possible to reduce quantity of data to be acquired, simplify equipment, etc.

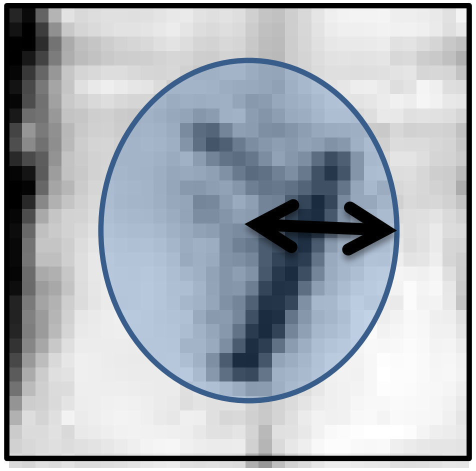
What do we want?

- To be a bit more quantitative, looking for high sensitivity of data to the parameters being sought

$$\frac{\partial\, \text{datum}_i}{\partial\, \text{pixel}_j} \text{ may be small}$$

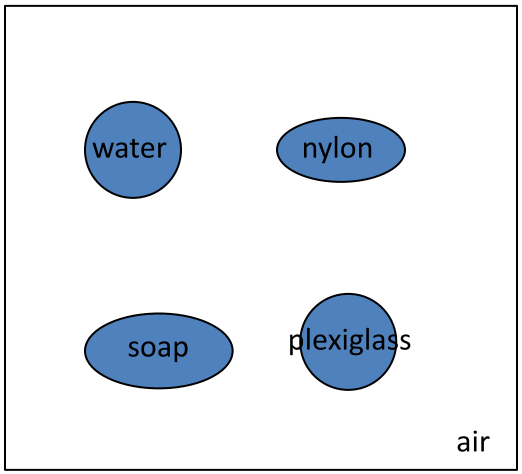
but

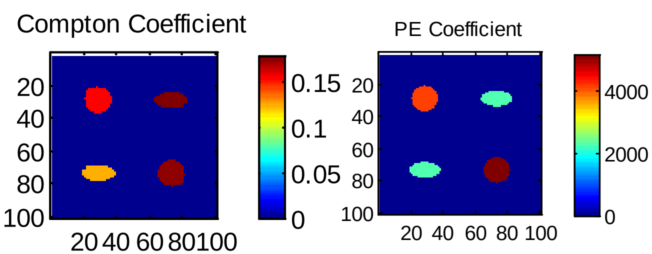
$$\frac{\partial\, \text{datum}_i}{\partial\, \text{radius}} \text{ may be large}$$



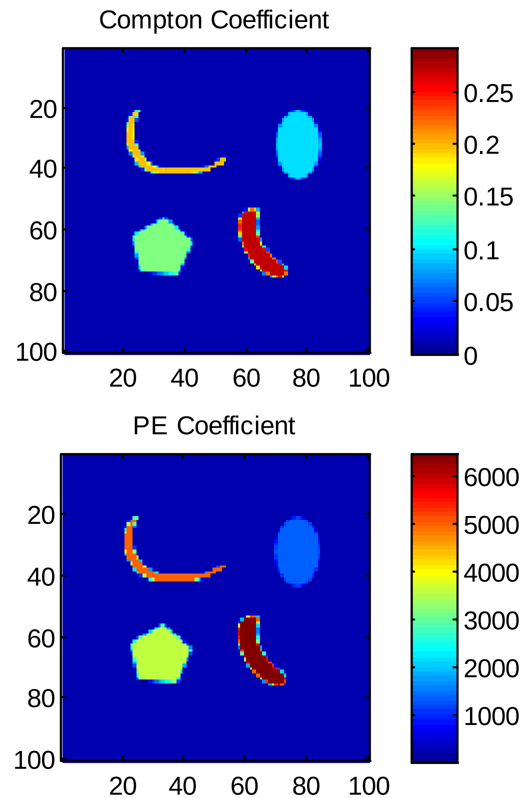
- Pixel-by-pixel approach “diffuses” information in the data across a huge number of unknowns
- More parametric methods may better concentrate the information to explain those degrees of freedom that actually are of most direct interest

Dual Energy CT Example



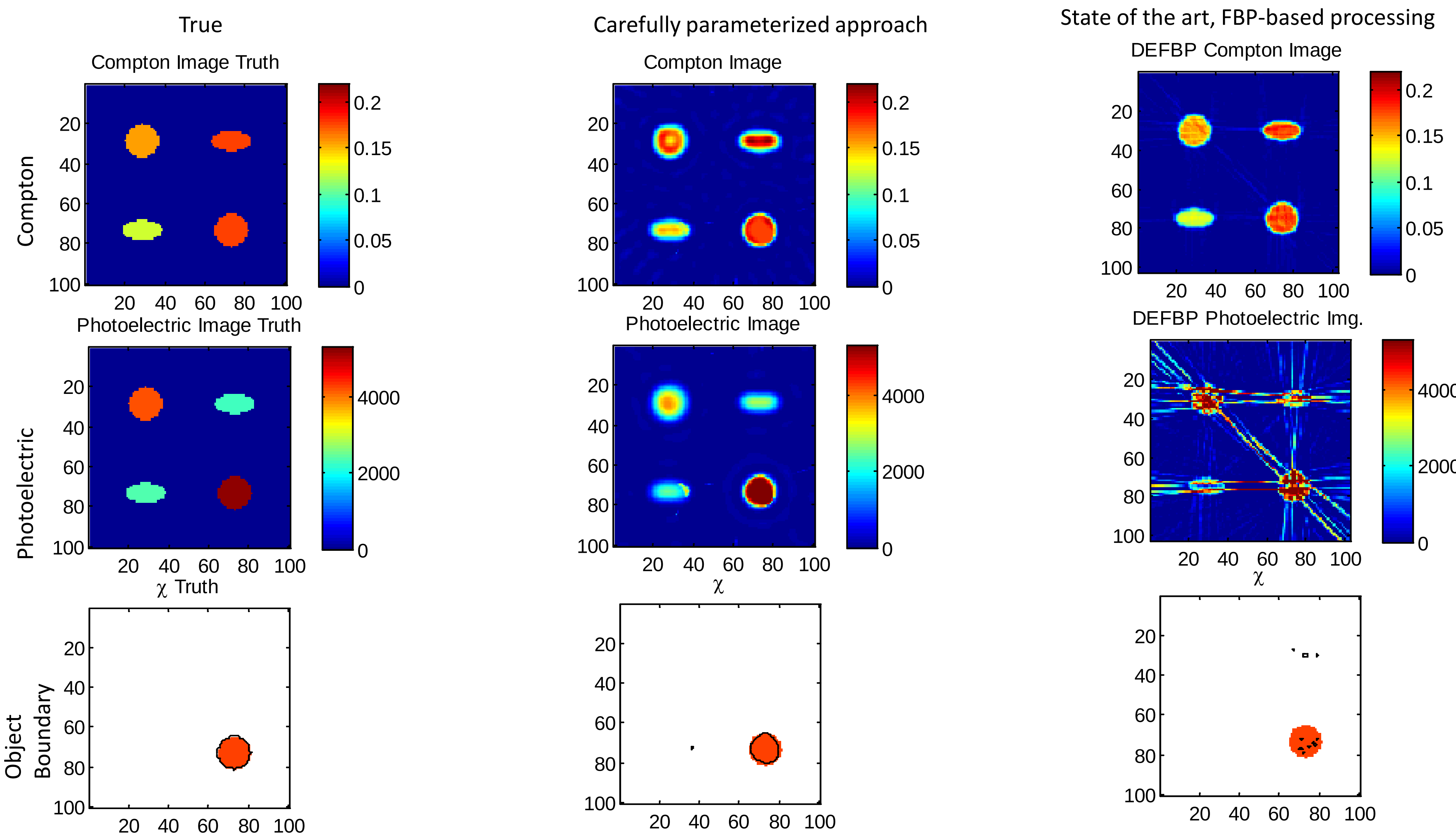


Object of interest: plexiglass

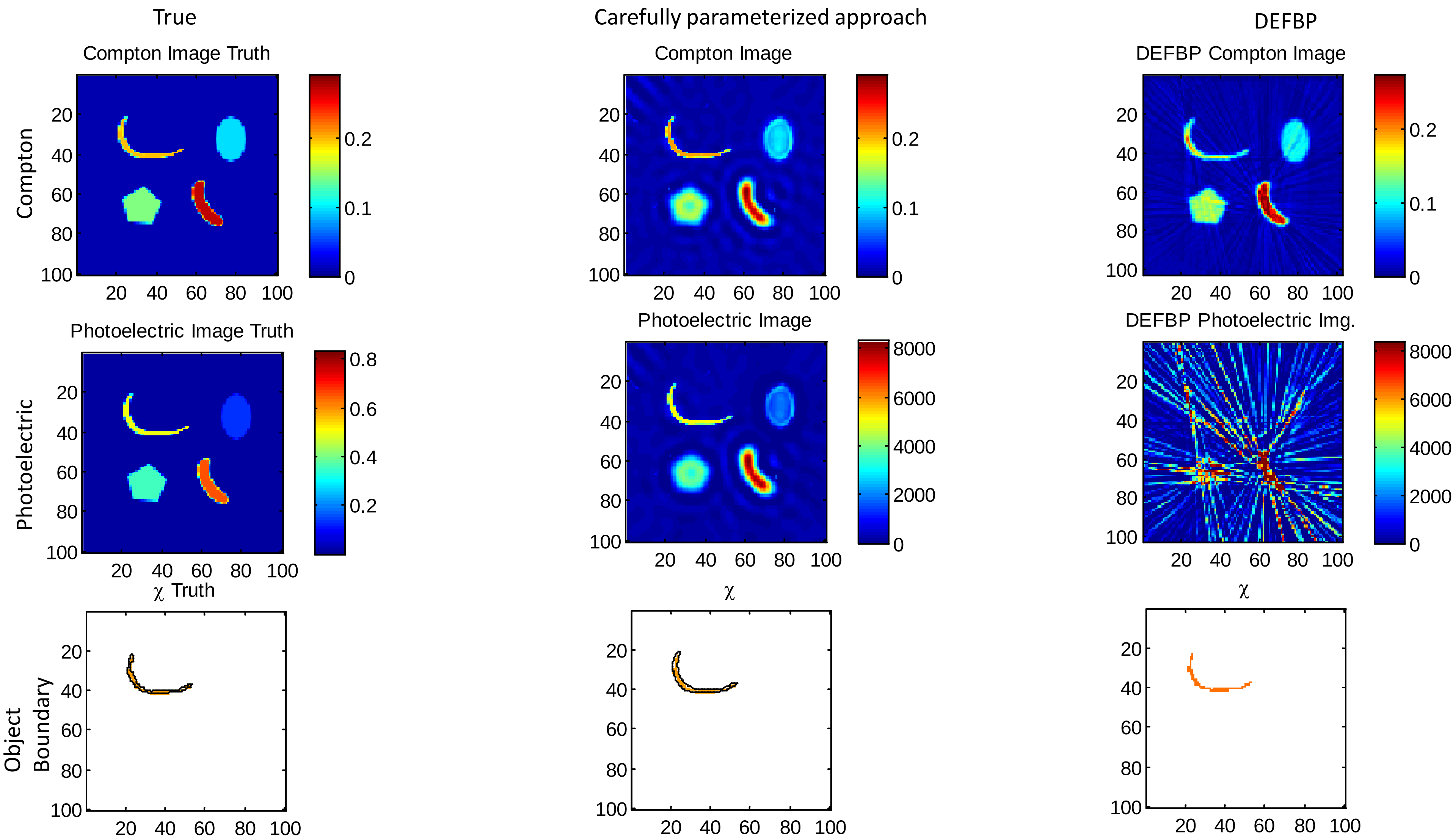


Object of interest: the thin yellow object

Dual Energy CT Results



More Results



Model Based Approach to Fusion

- Two sets of quantities
  - Data
  - Parameters
- Data:
  - Photon counts
  - Humidly levels
  - Voltages
  - Contaminant concentrations
- Parameters: derived from constitutive properties of the scene
  - Photoelectric or Compton scattering coefficients
  - Electrical permittivity and conductivity
  - Chemical concentrations
  - Contaminant saturation
- Models relate (1) data to parameters and (2) parameters to parameters

Data models

- From physics

$$d_i = f_i(x_i) \quad i = 1, 2, \ldots, N$$

- Many forms
  - Direct observation (*f* = identity)
  - Matrix equation
    - Spectral unmixing
  - Integral equation
    - Radon transform for CT
    - Kirchoff integral for some optics problems
  - Partial differential equation
    - Flow and transport
    - Electrical resistance tomography

Property Models

- There has to be *some* relationship among the x_i otherwise there is no fusion.
- A number of options or such models
  - Physics-based
  - Statistical
  - Geometric

Physics-based

- Petro-physical relationships
- Archie's law (electrical conductivity to porosity, saturation..)

$$\sigma = a\sigma_b\phi^m$$

- Complex refractive index method (dielectric to porosity and saturation)

$$\sqrt{\varepsilon} = (1-\phi)\sqrt{\varepsilon_s} + \phi S\sqrt{\varepsilon_w} + n(1-S)\sqrt{\varepsilon_a}$$

- Gassmann (seismic velocities to bulk/shear modulus, density)

$$V_p^2 = \frac{K + 4/3\mu}{\rho} \quad V_s^2 = \frac{\mu}{\rho}$$

Statistical Models

- Many, many options here
  - Lead to some type of maximum a posteriori or Bayesian approach to fusion
- Pairs of parameters are the same up to some noise

$$x_i = x_j + w_{i,j} \quad w_{i,j} \sim N\left(0, R_{i,j}\right)$$

- Parameters are jointly Gaussian

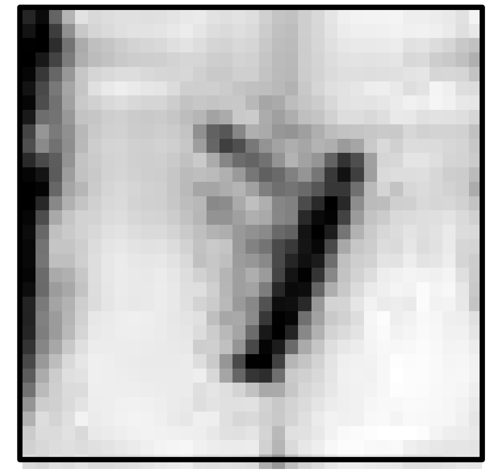
$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \sim N\left(\begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{bmatrix}, \begin{bmatrix} R_{1,1} & R_{1,2} & R_{1,3} \\ R_{2,1} & R_{2,2} & R_{2,3} \\ R_{3,1} & R_{3,2} & R_{3,3} \end{bmatrix}\right)$$

- Pairs of parameters have large mutual information

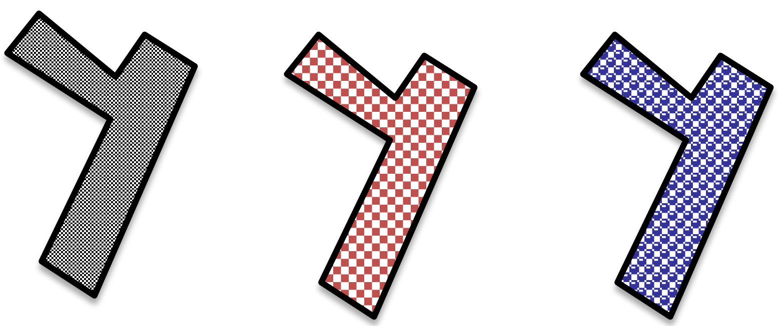
$$MI\left(x_1, x_2\right) = \sum p\left(x_1, x_2\right) \log \frac{p\left(x_1, x_2\right)}{p\left(x_1\right) p\left(x_2\right)}$$

Geometric Models

- An object in one property is an object in all properties



$$x_i(r) = c_i(r)\chi(r)$$



$x_1(r)$ $x_2(r)$ $x_3(r)$

Same shapes, χ, different contrasts c_i

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End Result

- Model based approach leads to variational methods for fusion

$$\hat{x}_1, \ldots, \hat{x}_N = \arg\min_{x_1, \ldots, x_N} \sum_{i=1}^{N} \left\| d_i - f_i(x_i) \right\|_2^2 + \Omega(x_1, \ldots, x_N)$$

Want to fit
to the data

Encourage similarity
based on property
models

- Other formulations possible as well depending on the models
- Structure leads to interesting and efficient algorithms
- Variational approach can be used for
  - Performance analysis
  - Evaluation of information content of data sources
  - Optimization of data collection or instrument design
  - Etc

Example

Electrical Model

$$d_E = M_E(\sigma)$$

Hydrological Model

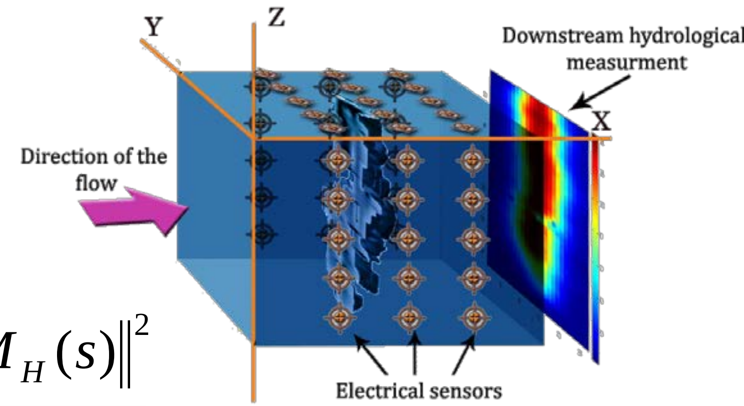
$$d_H = M_H(s_{NAPL})$$

Petrophysical Models, *Hunt's* Model, or *Archie's* Law

$$\sigma = T(s_{NAPL})$$

$$s_{NAPL} = \arg\min_s \left\| d_E - M_E\left(T(s)\right) \right\|^2 + \lambda \left\| d_H - M_H(s) \right\|^2$$

$$s = s_i H\left(\phi(\alpha,\beta,\chi)\right) + s_0\left(1 - H\left(\phi(\alpha,\beta,\chi)\right)\right)$$



$$\begin{cases} \arg\min_s \left\| d_E - M_E\left(T(s)\right) \right\|^2 \\ \arg\min_s \left\| d_H - M_H(s) \right\|^2 \end{cases}$$

Forward Models

Poisson's Equation

$$\nabla \cdot \left(\sigma(\mathbf{x}) \nabla v(\mathbf{x}) \right) = i(\mathbf{x})$$

$\sigma(\mathbf{x})$ **Electrical Conductivity**

$i(\mathbf{x})$ **Current Source Distribution**

$v(\mathbf{x})$ **Electrical Potential**

Mass Transport Equation

$$\frac{\partial}{\partial t}\left(\theta_w C_i\right) + \nabla \cdot \left(C_i \vec{q} - \theta_w \mathbf{D}_H^i \cdot \nabla C_i \right) = E_{an_i}$$

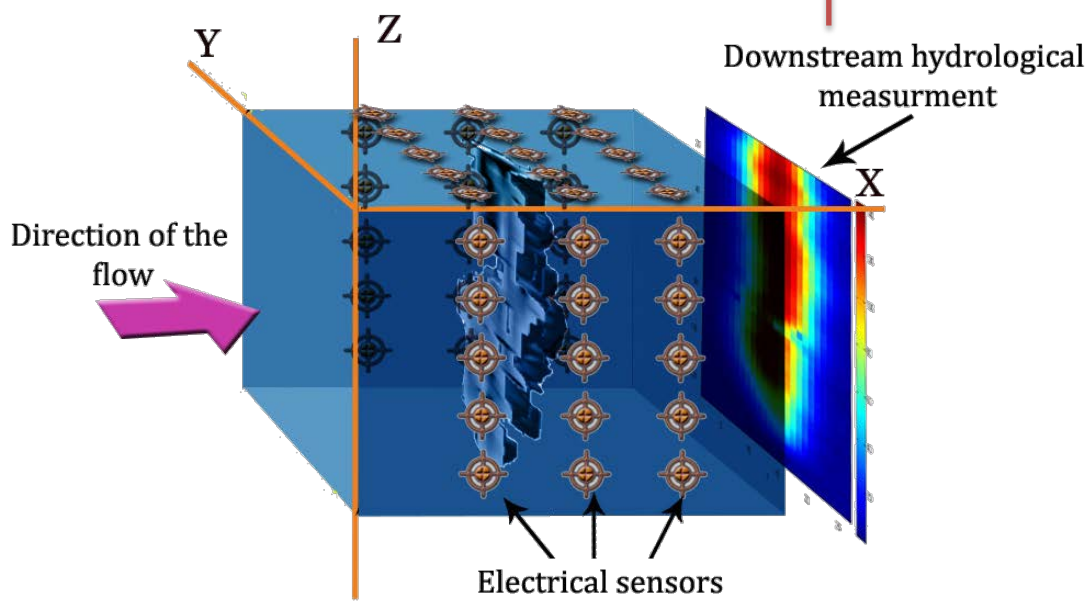
θ_w **Volumetric Water Content**

C_i **Mass Concentration of Component i**

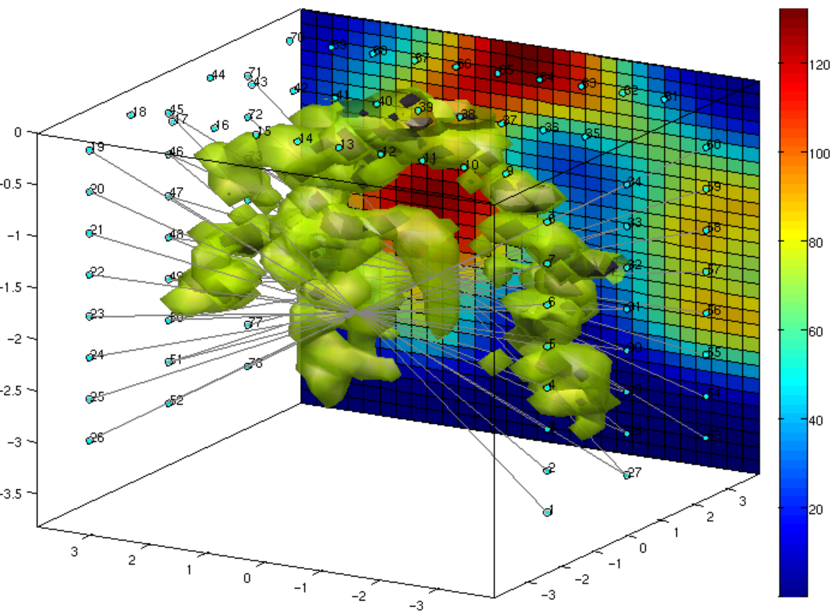
$\vec{q}$ **Specific Discharge from Darcy's Law**

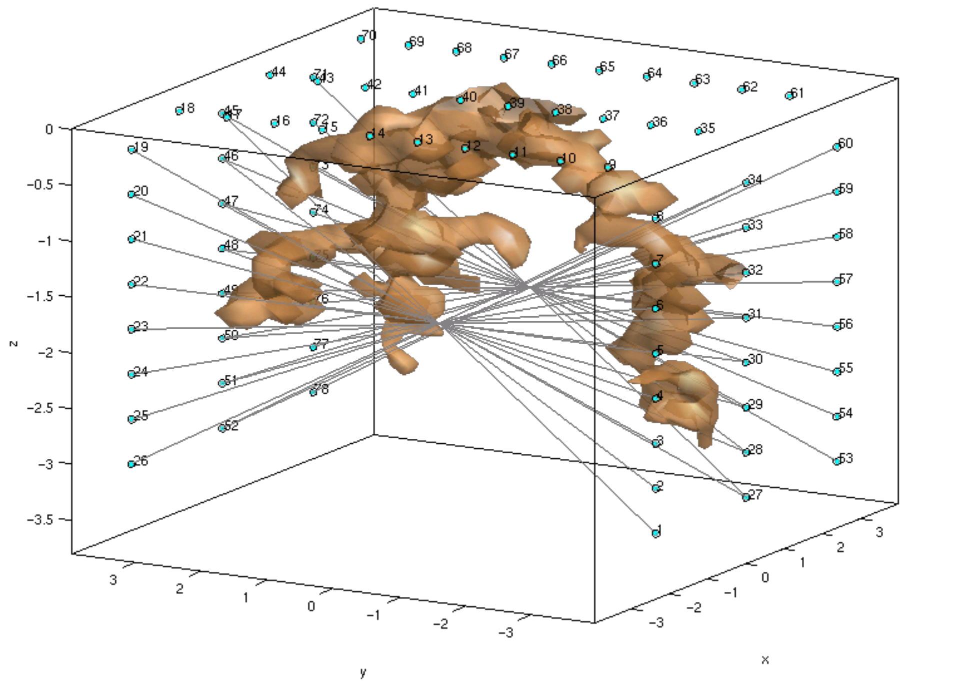
$\mathbf{D}_H^i$ **Hydrodynamic Dispersion Tensor**

E_{an_i} **The Interphase Mass Exchange of Component i from the NAPL to aqueous phase**



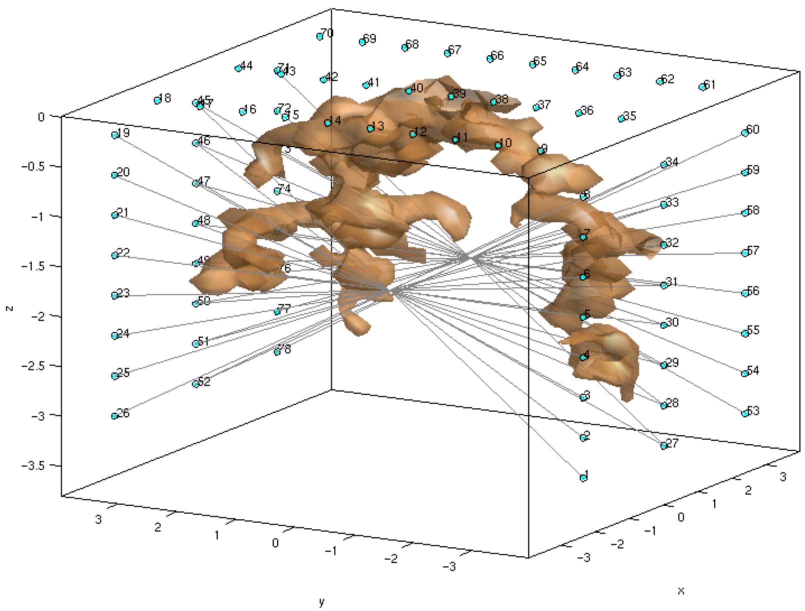
Simulations

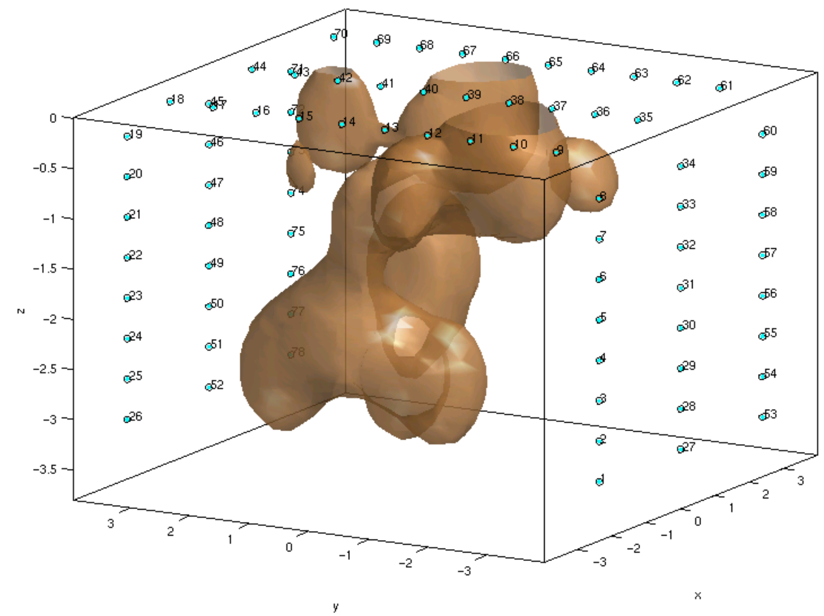




Left: Original Saturation Profile at 1% and 15% , Right: Original Saturation Profile at 1% only

Initialization

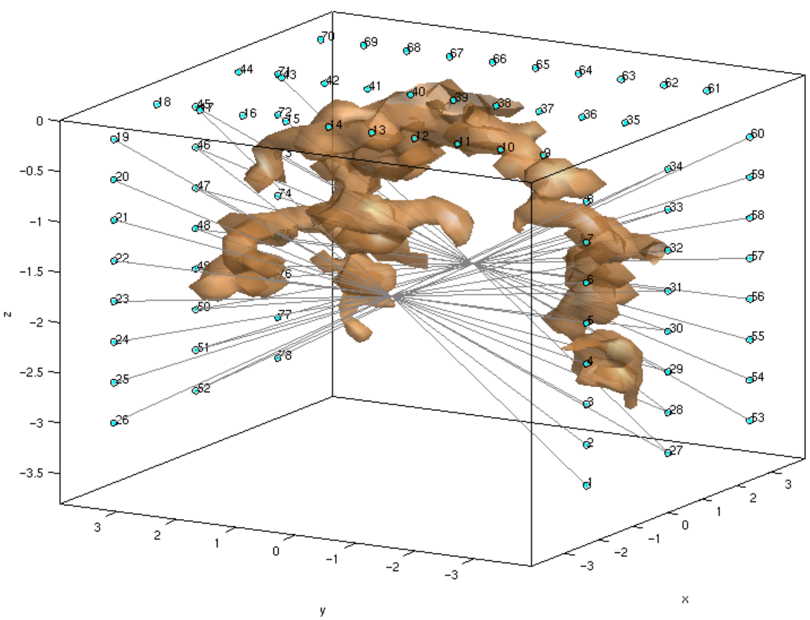




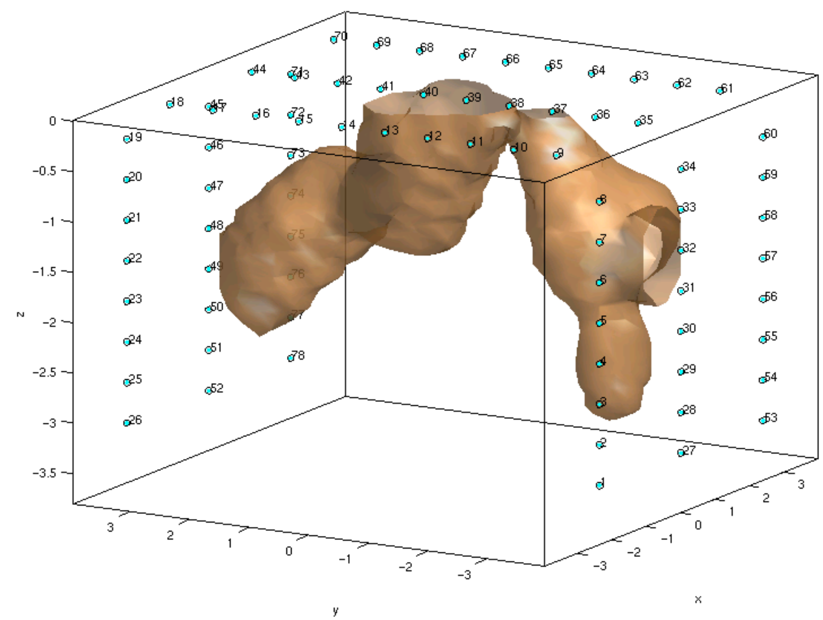
Original

Initial shape. Initial saturation = 1%

Results

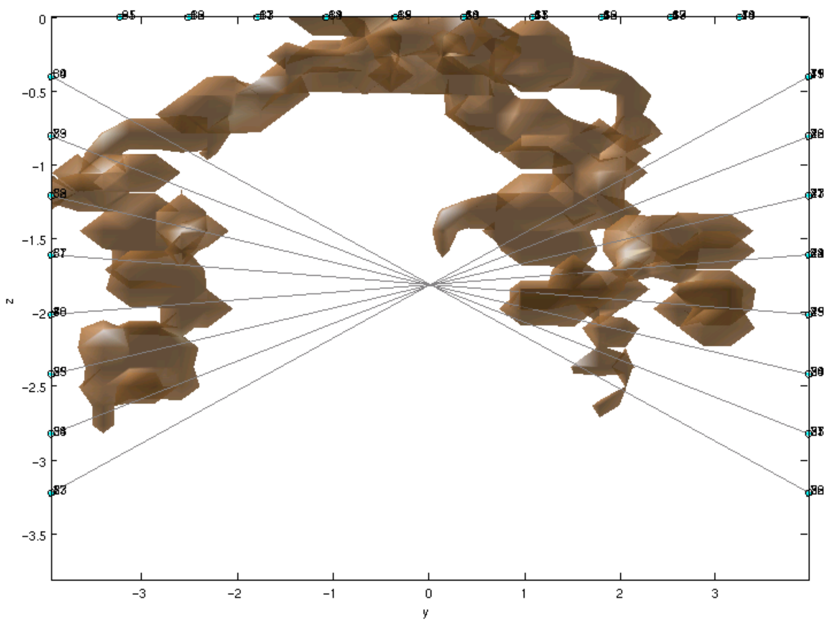


Original

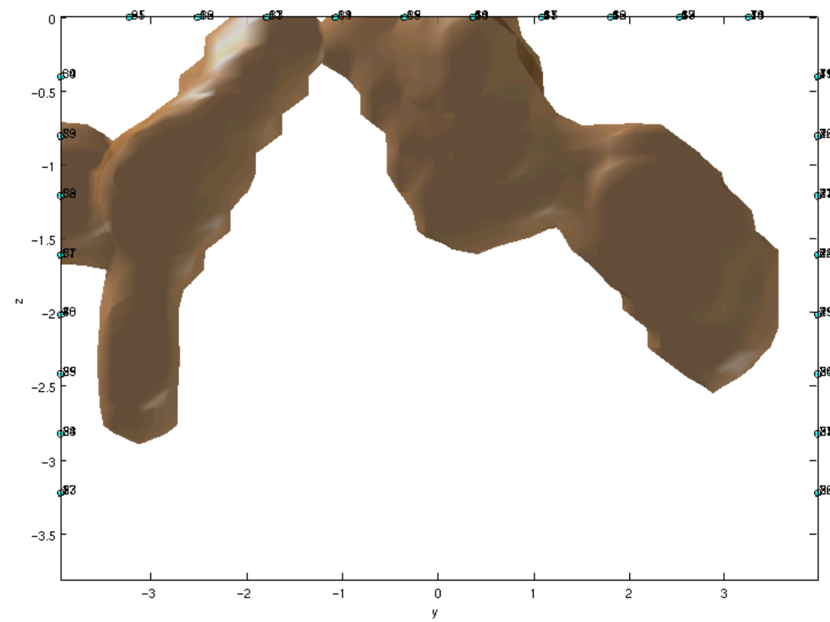


Final reconstruction, final saturation value= 2.92%

Results

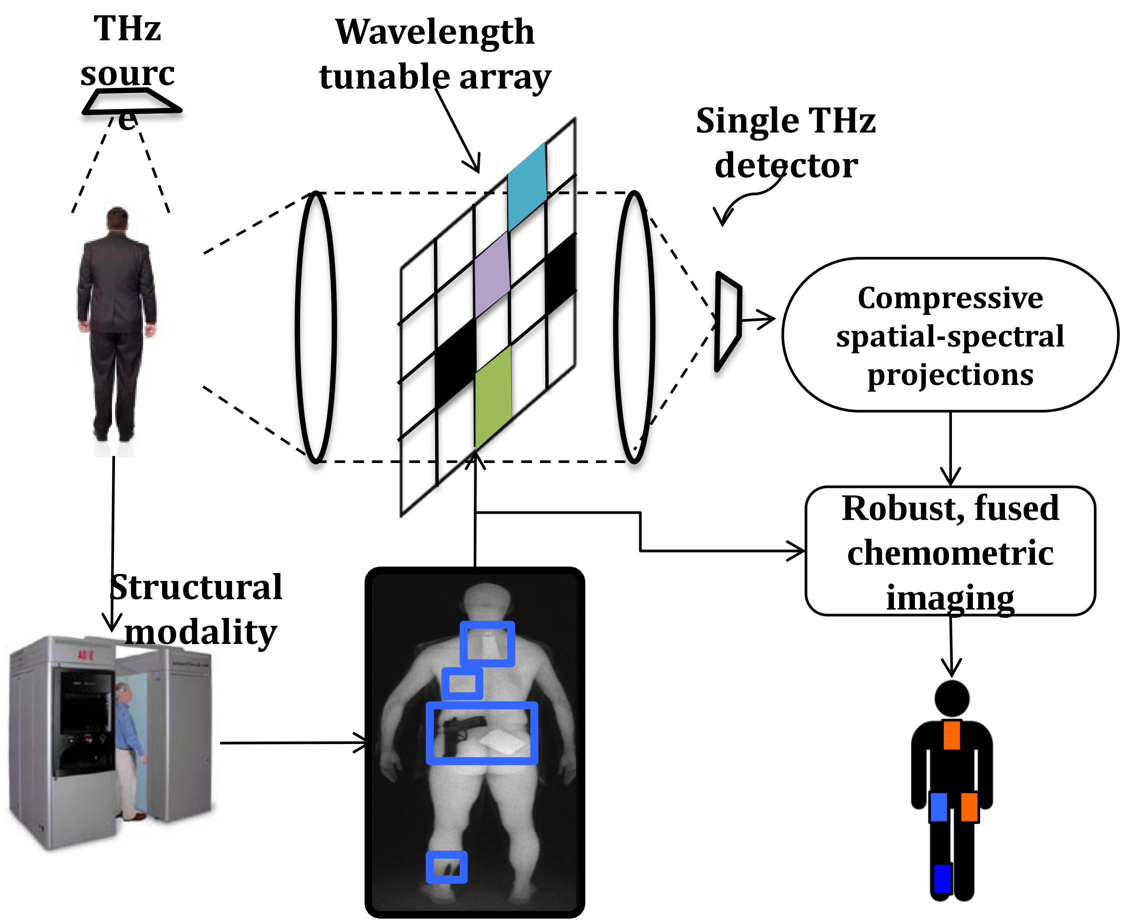


Original



Reconstruction side view

Security Application



- Currently considering THz spectroscopy + structural modality(ies)
- Proposal under review at DHS involving team from Tufts (Miller, Tracey, Sonkulsale, Aeron) BC (Padilla), and Kaiserslautern (Rahm and Beigang)
- Characteristics
  - Tight integration of instruments and processing
  - Model based (physics sensor models and statistical/information theoretic property models)
  - Extensive experimental component

Conclusions

- Model based methods have much to offer for fusion
- Principled approach to many issues associated with multi-sensor data acquisition, processing, and analysis
- Physics-based models allow for joint design and optimization of sensors and processing
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