System Design Considerations for CAXSI:

Coded Aperture X-ray Scatter Imaging

Joseph A. (Jody) O'Sullivan

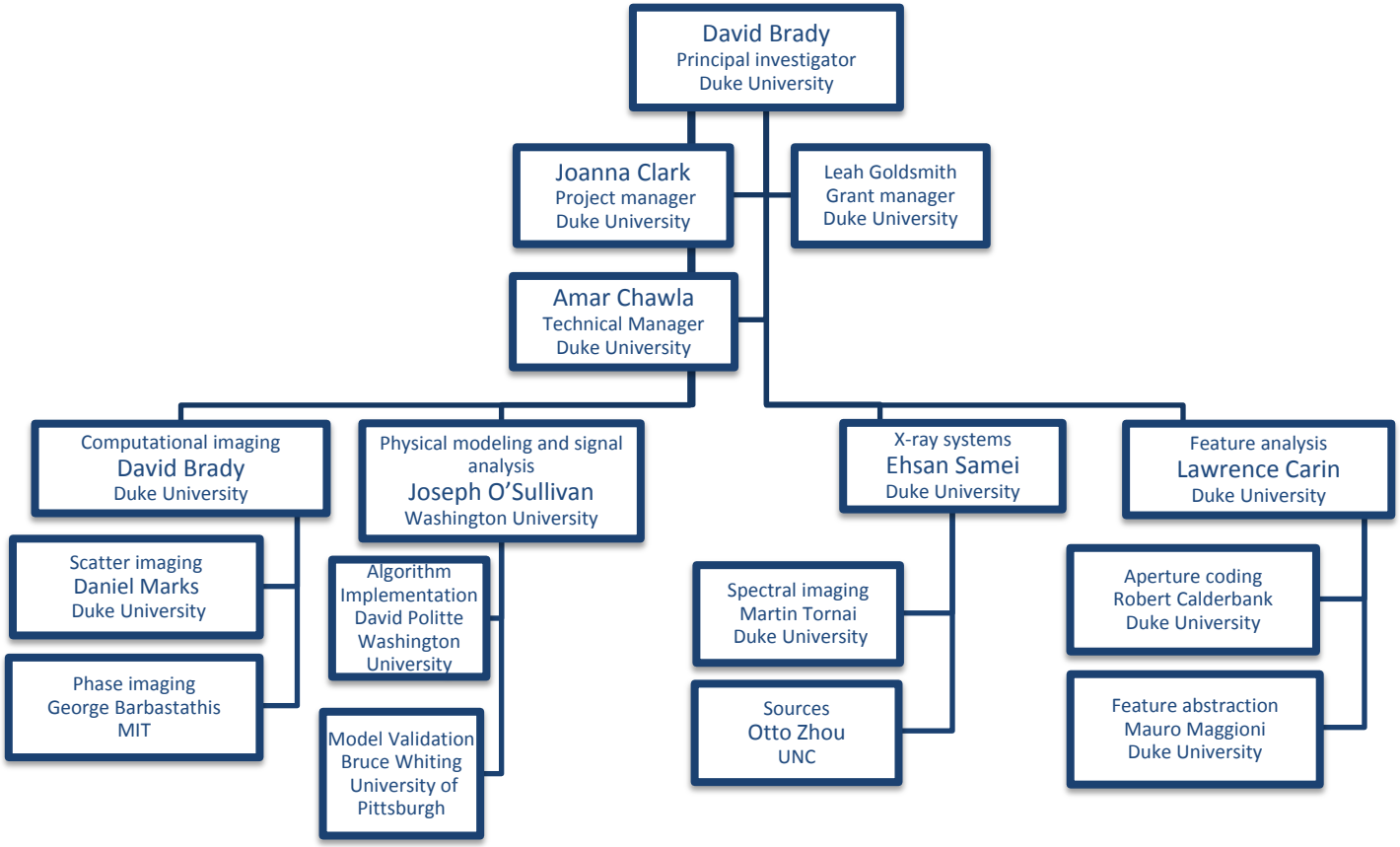
and the CAXSI Team

DHS S&T: HSHQDC-11-C-00083

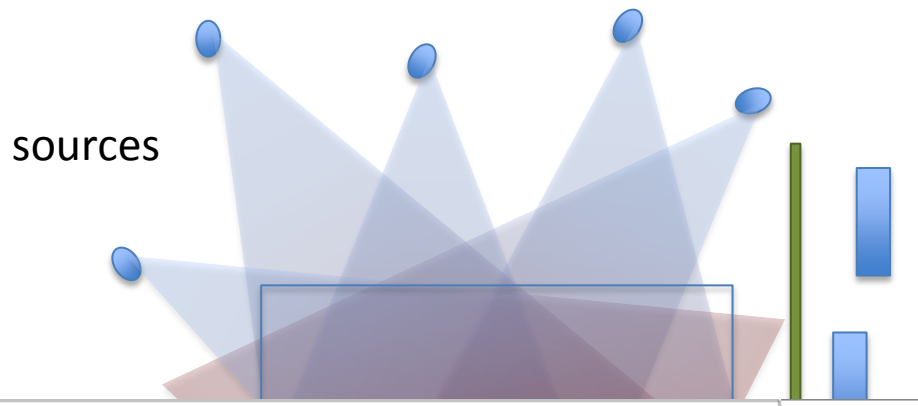
CAXSI Team

- David Brady, Larry Carin, Robert Calderbank, Amarpreet Chawla, Anuj Kapadia, Kalyani Krishnamurthy, Andrew Holmgren, Pooyan Bagherzadeh, Ehsan Samei, Martin Tornai, Mauro Maggioni, Randy McKinley, Scott Wolter, Duke University
- Jody O'Sullivan, David Politte, Ikenna Odinaka, Washington University
- Bruce Whiting, University of Pittsburgh
- Otto Zhou, Kenneth MacCabe, UNC
- George Barbastathis, Jon Petrocelli, Lei Tian, MIT

CAXSI Team

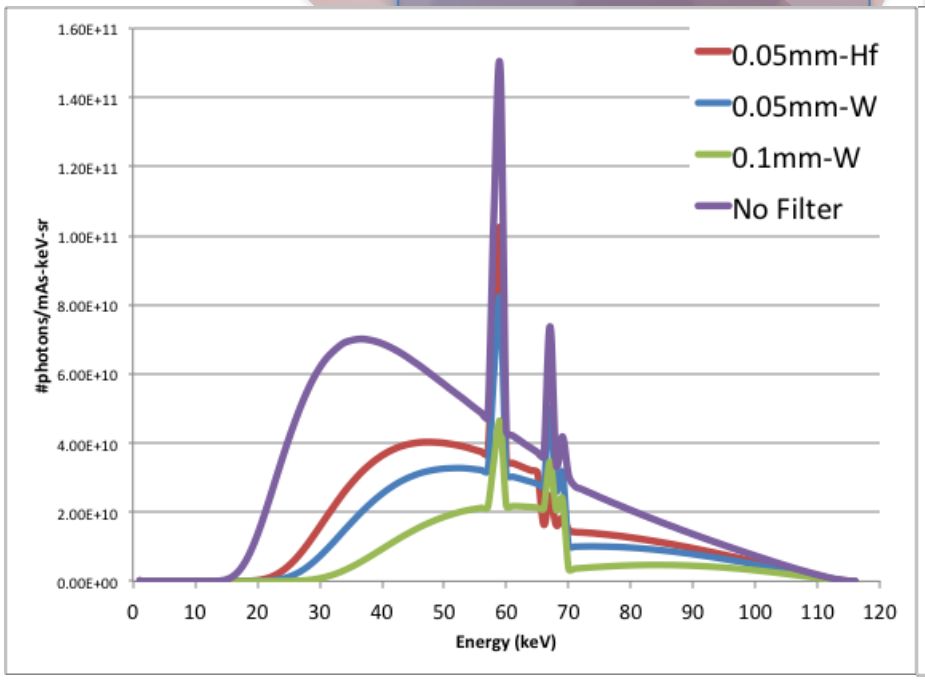

David Brady
Principal investigator
Duke University
Joanna Clark
Project manager
Duke University
Leah Goldsmith
Grant manager
Duke University
Amar Chawla
Technical Manager
Duke University
Computational imaging
David Brady
Duke University
Physical modeling and signal analysis
Joseph O'Sullivan
Washington University
X-ray systems
Ehsan Samei
Duke University
Feature analysis
Lawrence Carin
Duke University
Scatter imaging
Daniel Marks
Duke University
Phase imaging
George Barbastathis
MIT
Algorithm Implementation
David Politte
Washington University
Model Validation
Bruce Whiting
University of Pittsburgh
Spectral imaging
Martin Tornai
Duke University
Sources
Otto Zhou
UNC
Aperture coding
Robert Calderbank
Duke University
Feature abstraction
Mauro Maggioni
Duke University

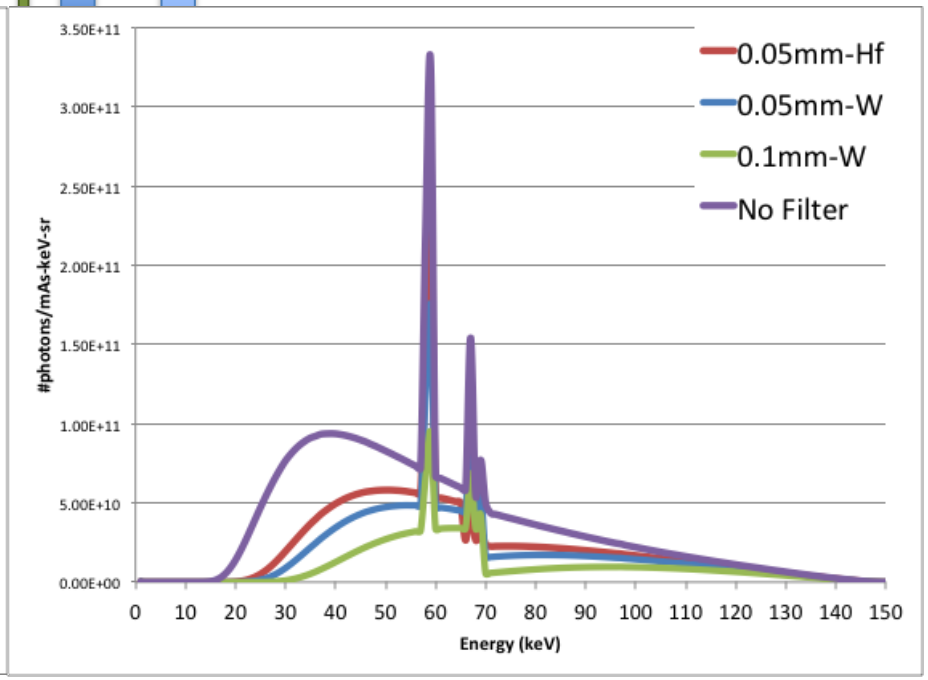
CAXSI System Vision



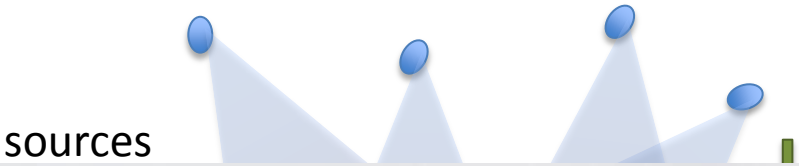
Selected Key Components

- Distributed sources
  - Novel sources
  - Spectra
  - Primary aperture
- Various detectors
- Coded aperture(s)

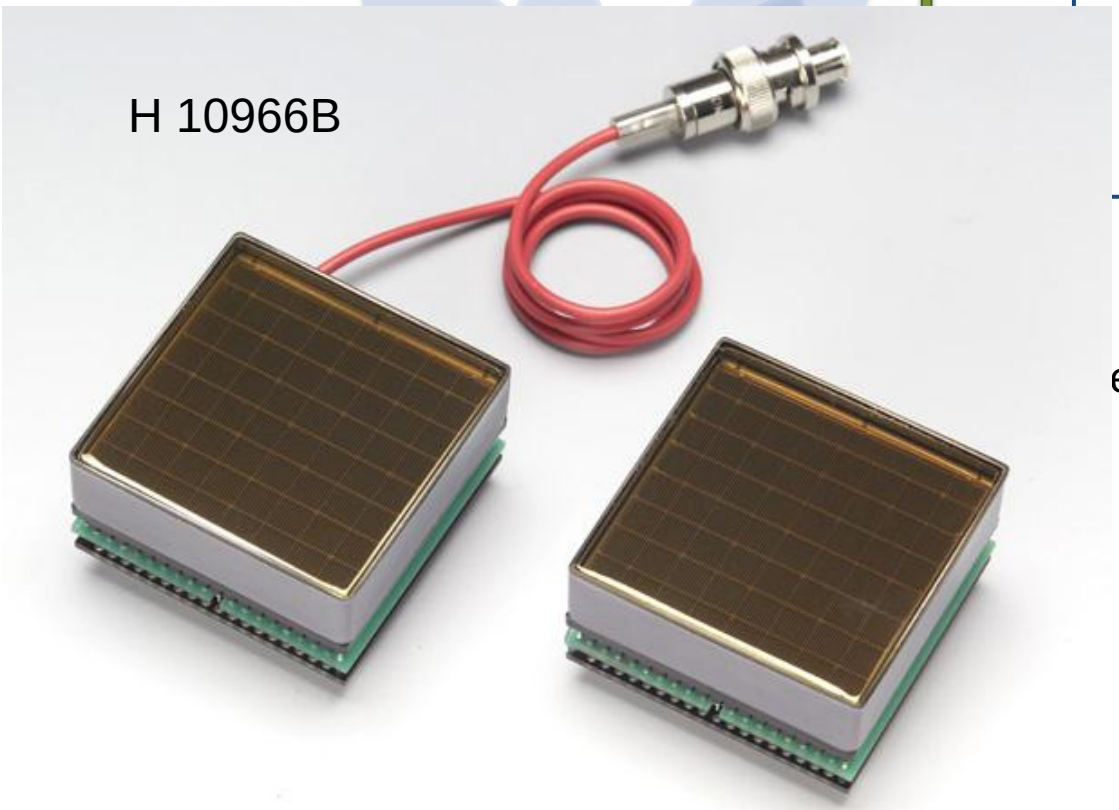




CAXSI System Vision



sources



ectors

Selected Key Components

- Distributed sources
  - Novel sources
  - Spectra
  - Primary aperture

Various detectors

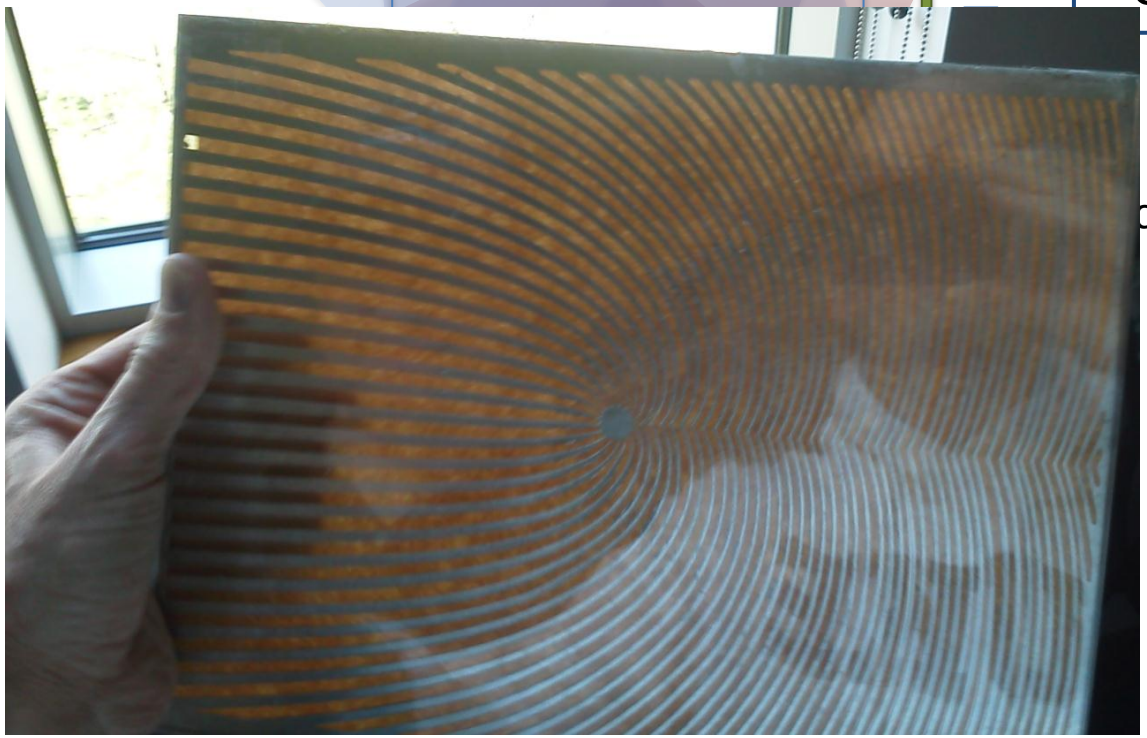
Coded aperture(s)

From *http://sales.hamamatsu.com/en/products/electron-tube-division/detectors/photomultiplier-tubes/part-h10966b.php*

CAXSI System Vision

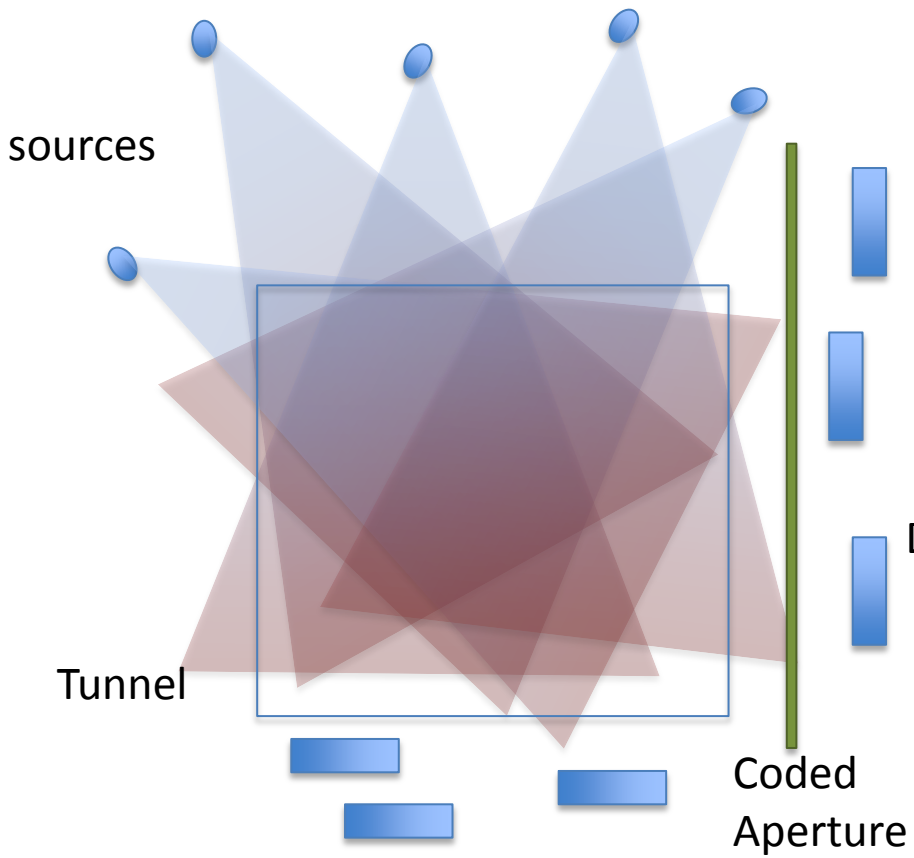
Selected Key Components

- Distributed sources
  - Novel sources
  - Spectra
  - Primary aperture
- Various detectors
- Coded aperture(s)



ctors

CAXSI System Vision



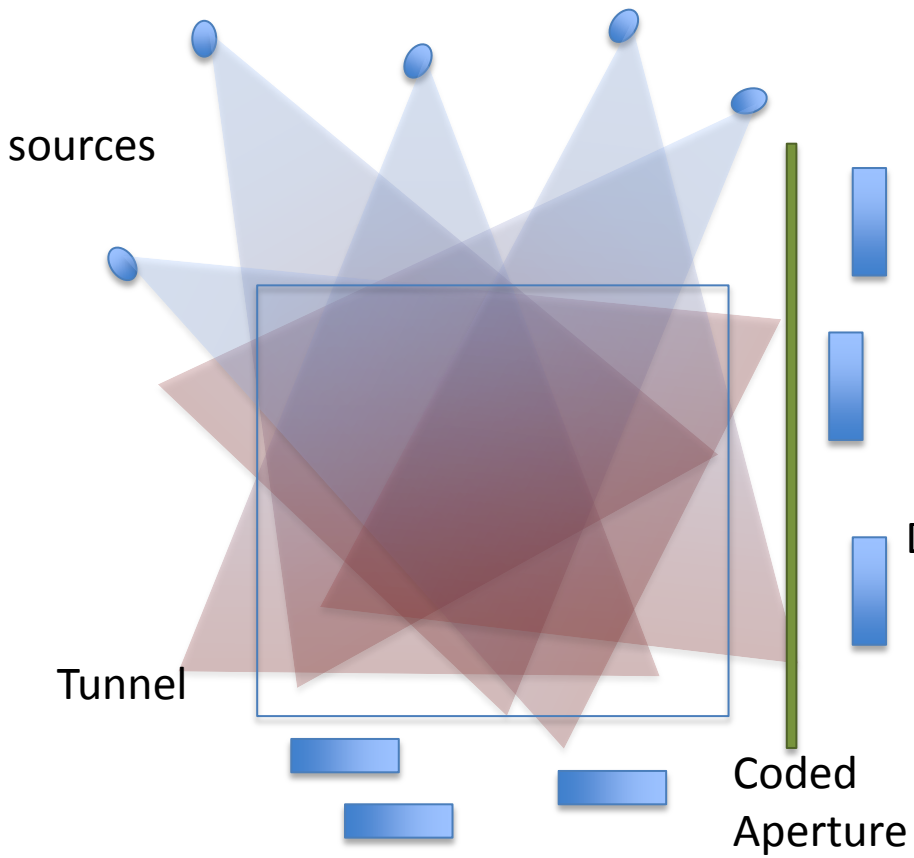
Selected Key Components

- Distributed sources
  - Novel sources
  - Spectra
  - Primary aperture
- Various detectors
- Coded aperture(s)

Selected Key Ideas

- Physical modeling of signals
- Signature characterization
- System design motivated by integrated sensing and processing (compressive sensing)
- Integration of components

CAXSI System Vision



Selected Key Limitations

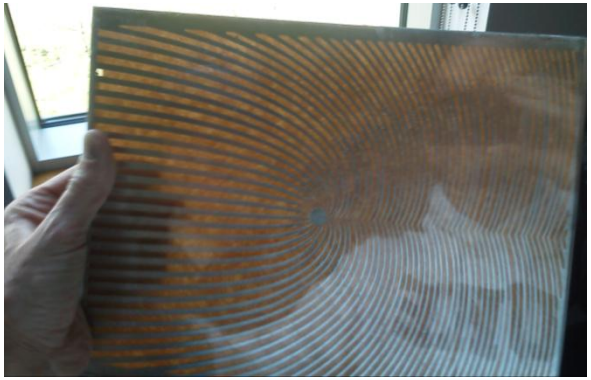
- Source spectral width
- Energy sensitivity of detectors
- Spatial extent of targets
- Unknown clutter in the luggage
- Low signal

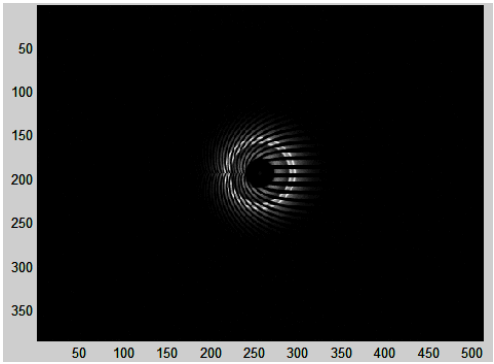
Selected Key Ideas

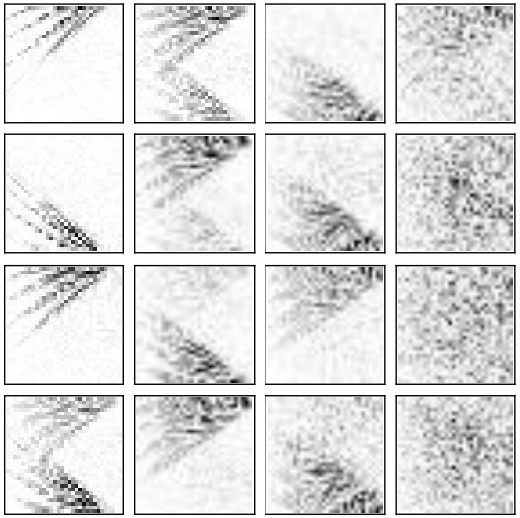
- Design system to increase sensitivity and specificity → Overcome blurring effects, optimally measure photons
- Multifaceted design space

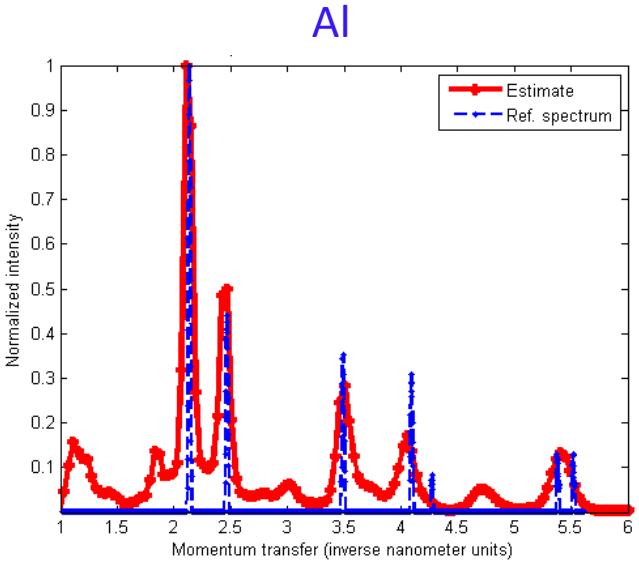
CAXSI Outline

- CAXSI System Vision
- Signature Analysis
  - Measurement space signature
    Forward models
  - Object space signature
    Reconstruction
  - Logical space signature
    SVD
- Conclusion

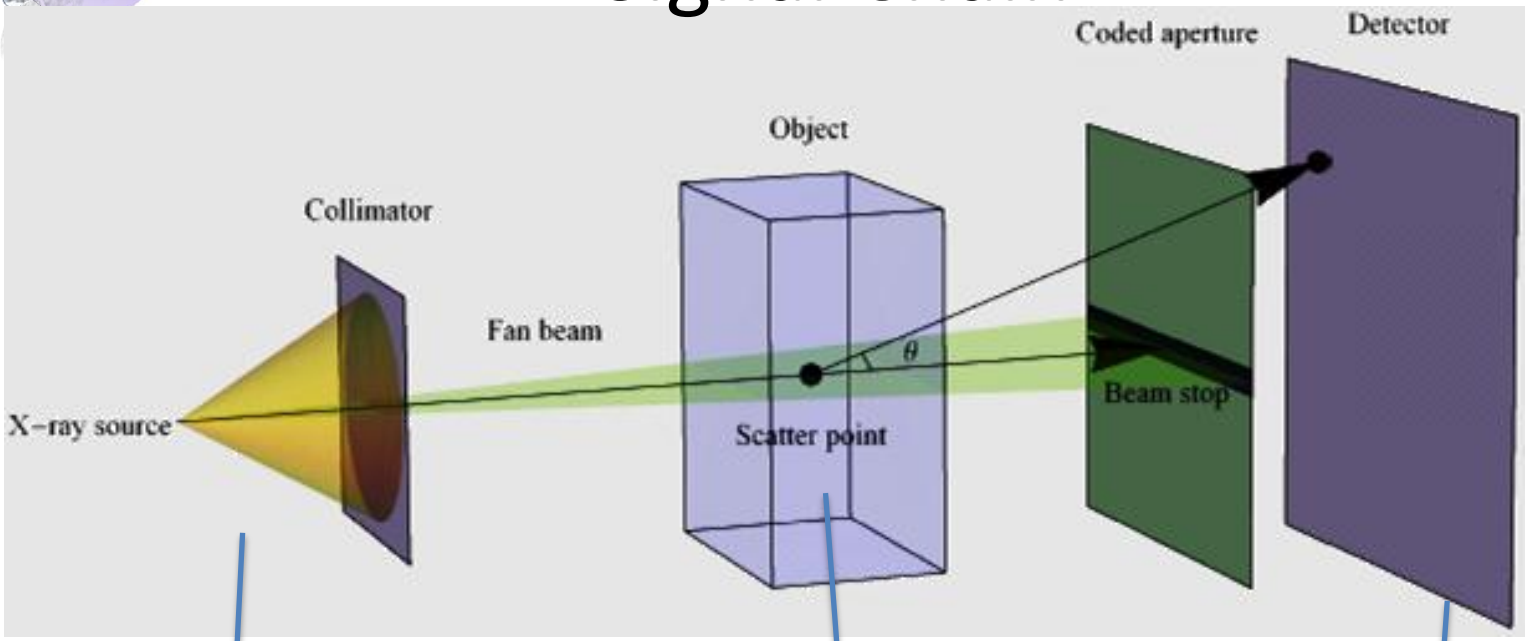


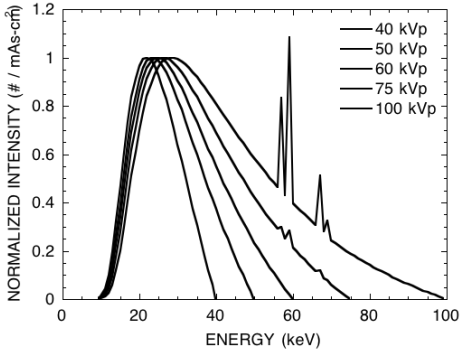


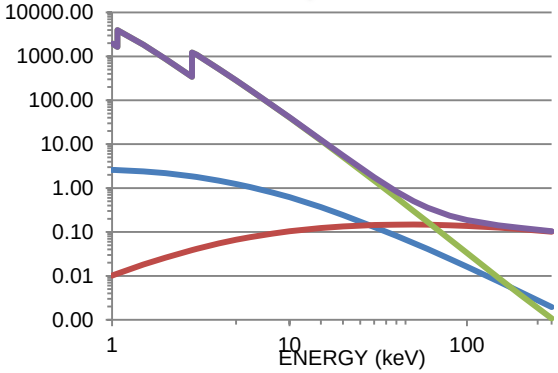


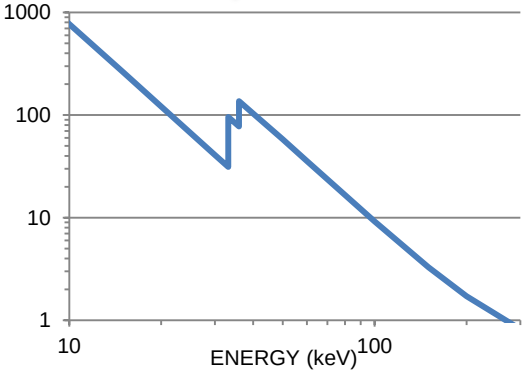


Signal Chain





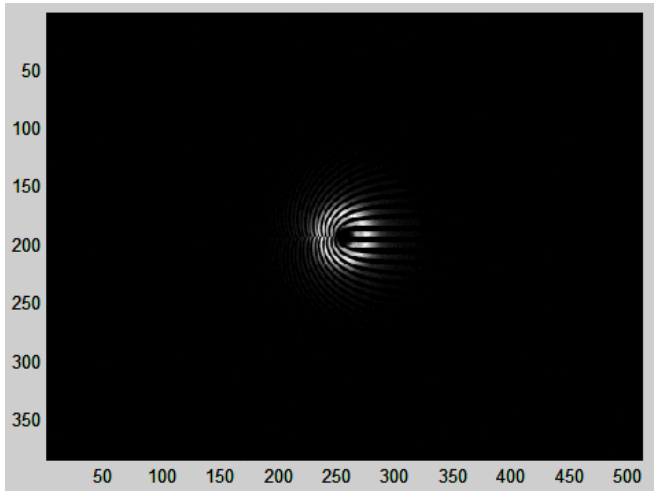


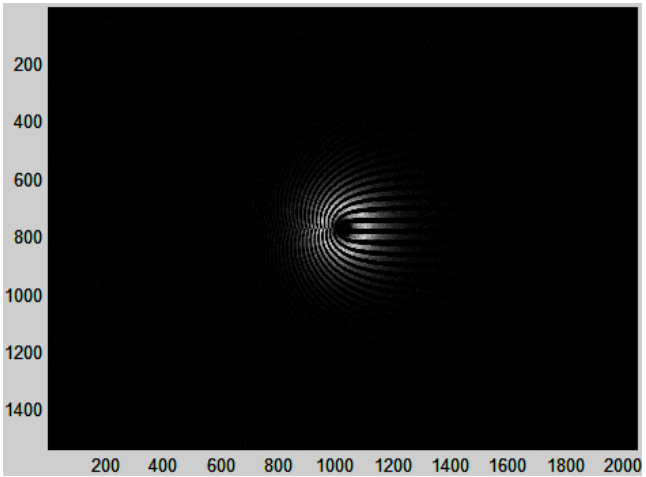


Signature Definition

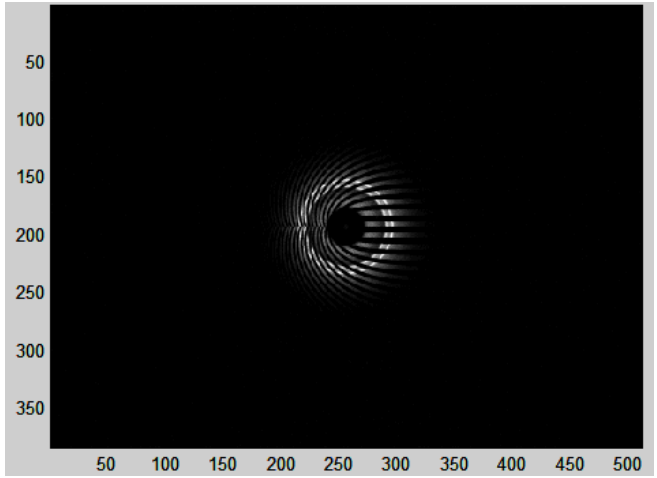
- Underlying characteristic of a target of interest under X-ray illumination
  - Employed to identify specific targets
  - Coherent scatter, incoherent scatter, attenuation
- Defined in three different spaces
  - Measured (Detector or measurement space)
  - Reconstructed (Target or object space)
  - Compressed (Logical or abstract space)
- Measured and reconstructed are acquired via experiments and/or MC simulations
- Compressed acquired via system design and integration

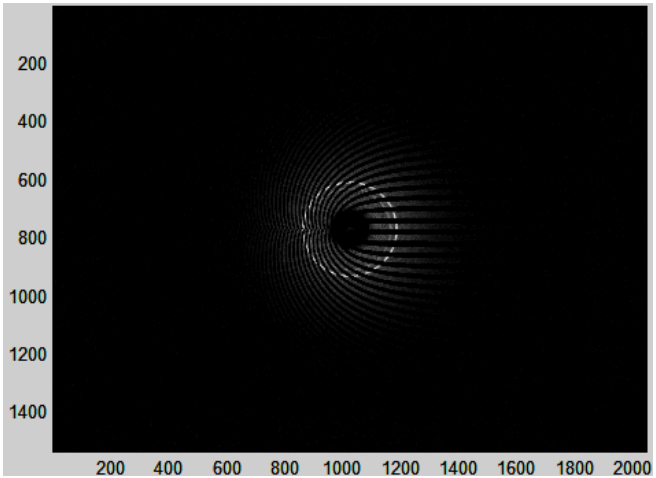
Example Signatures – Measurement Space (pencil beam, target alone)





Acrylic





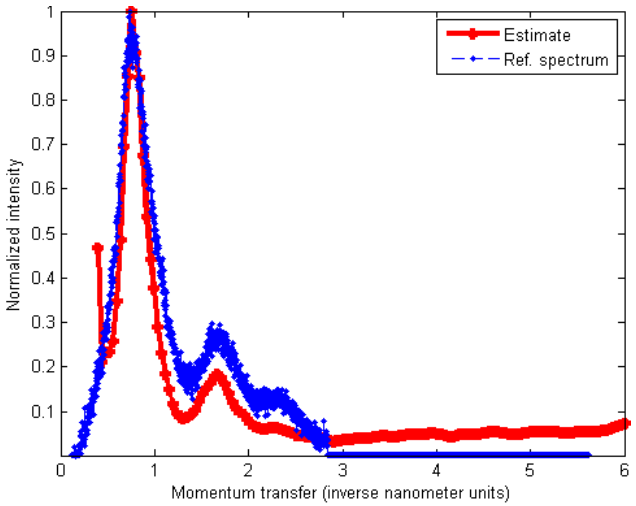
Graphite

150kVp, 0.1mm W

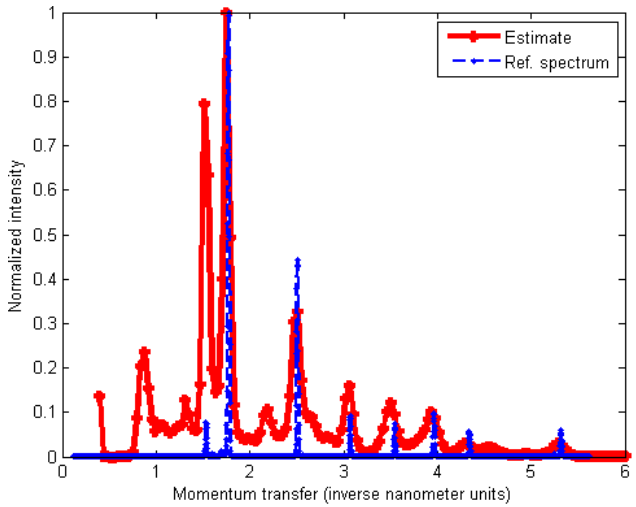
150kVp, No Filter

Example Signatures – Target Space (150kVp, 0.1mm W)

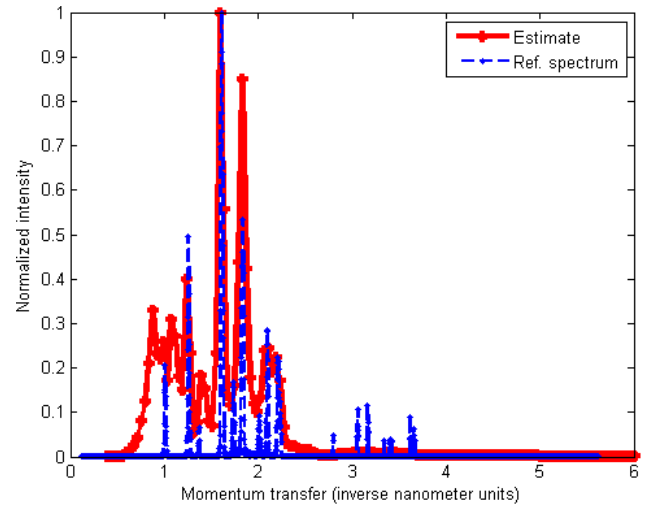
Acrylic



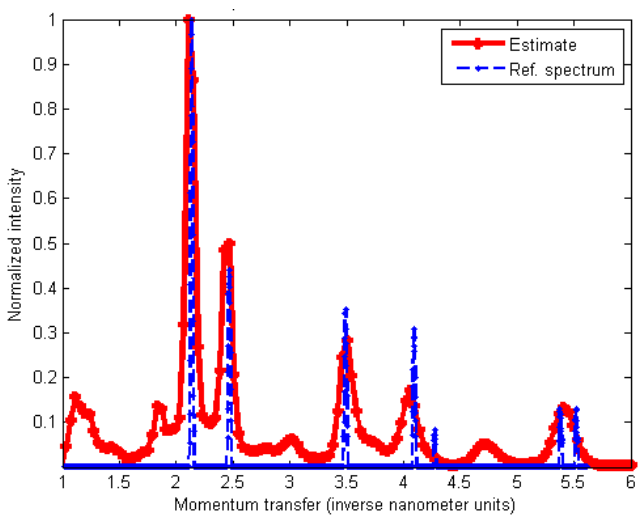
NaCl



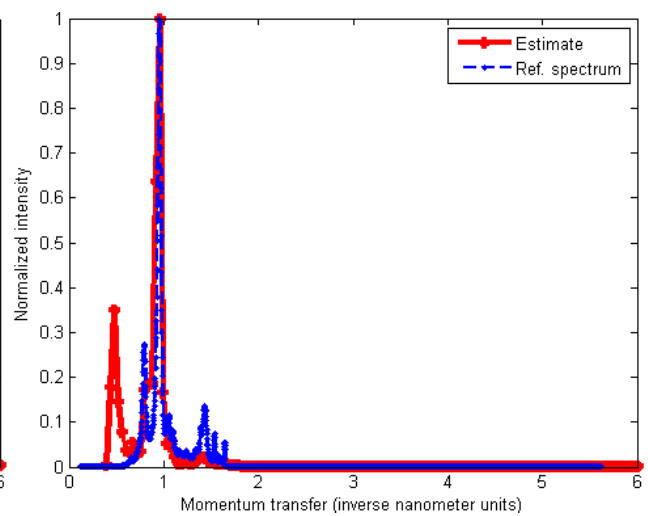
NH4NO3



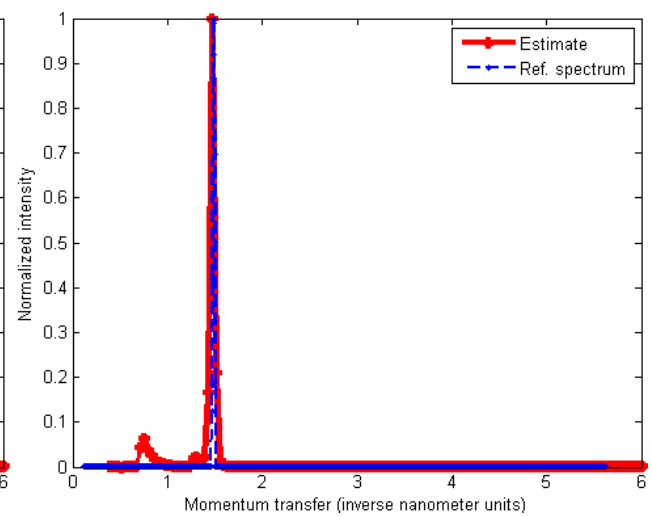
Al



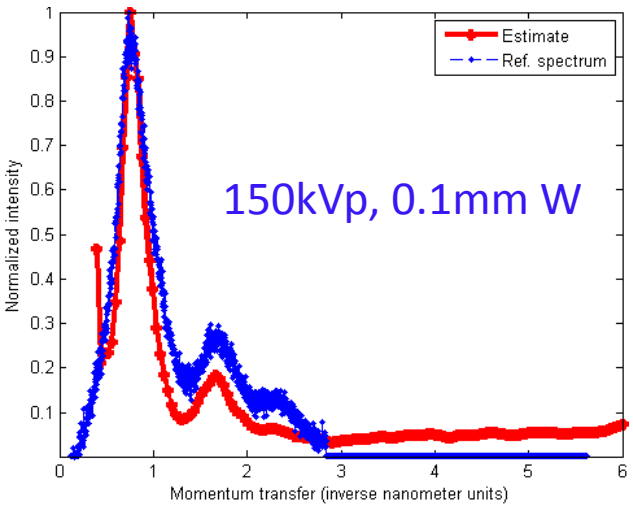
Milk chocolate

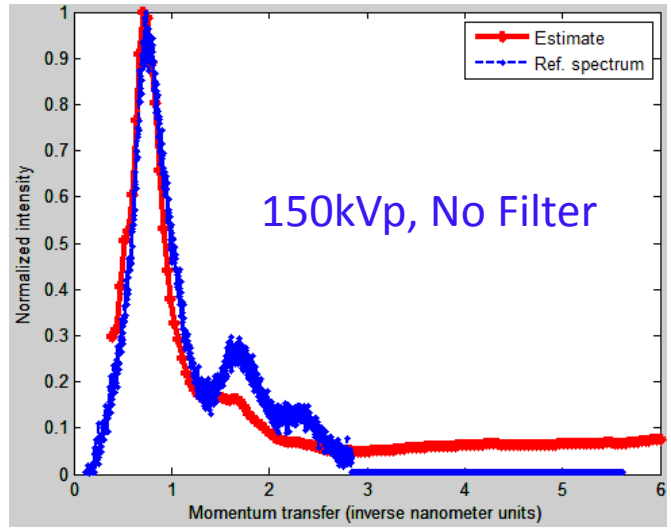


Graphite

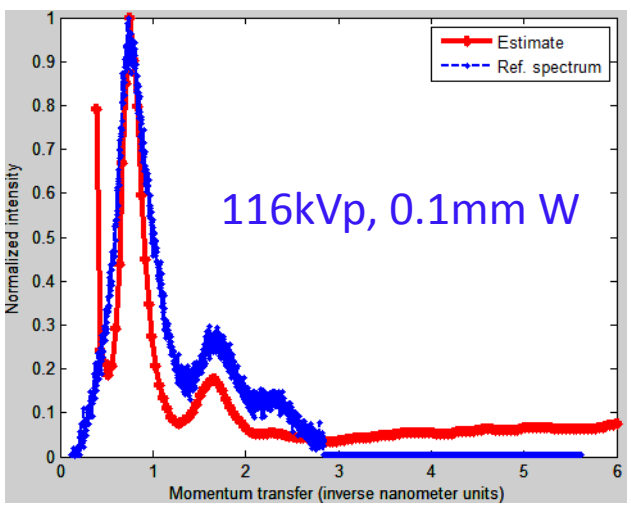


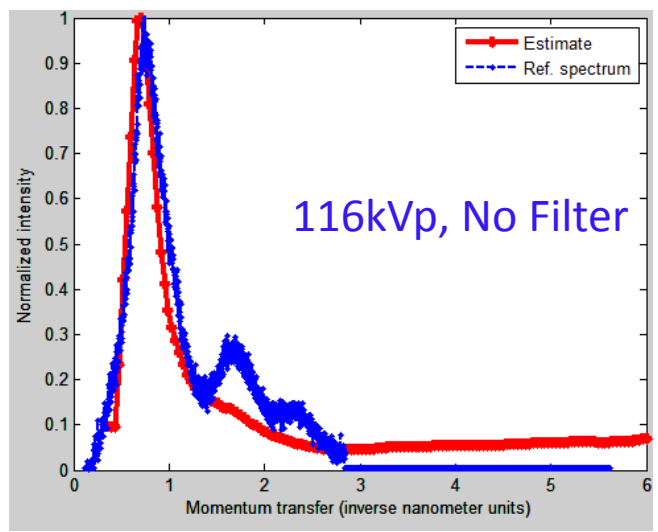
Reconstructed signature with different spectra



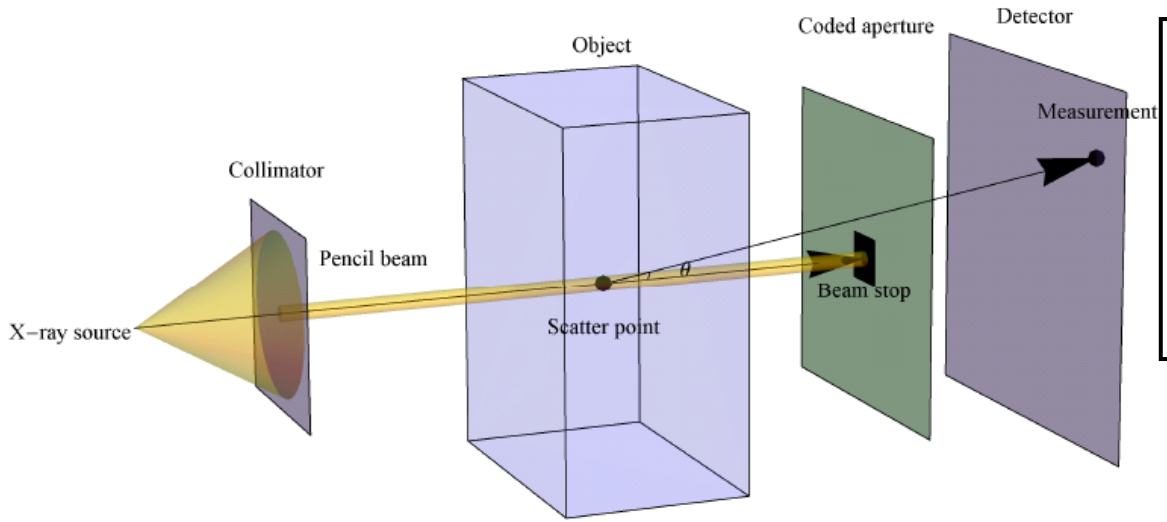


Acrylic

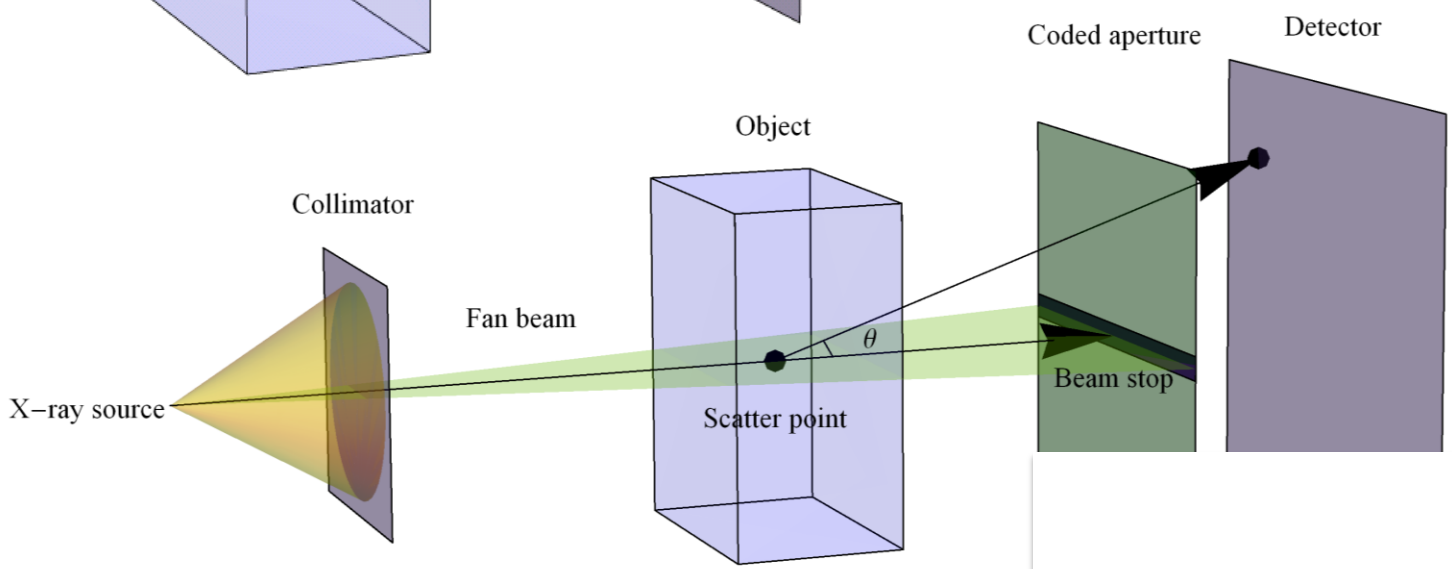




Pencil Beam → Fanbeam Model



$\mathbf{r} = [x, y, 0]$,	object point
$\mathbf{r}' = [X_d, y', z']$,	detector point
θ,	scatter angle
$\mathbf{s} = \mathbf{r}' - \mathbf{r}$,	scatter vector



Physics-Based Model

- Based on a radiance model, propagated using ray projection
- Objects have scattering densities f at each spatial location $\mathbf{r}$, as a function of momentum transfer q
- For coherent scatter at angle θ, Bragg's Law gives $q=2k\sin(\theta/2)$
- Given vector $\mathbf{s}$ from scattering point to detector whose normal is $\mathbf{n}$, there is a geometric factor $\frac{|\mathbf{n}\cdot\hat{\mathbf{s}}|}{s^2}$, where $\mathbf{s}=\mathbf{r}'-\mathbf{r}$
- Mask factor $T(\mathbf{r},\hat{\mathbf{s}})$
- Detector response $g(\mathbf{r}')$ in terms of impulse response

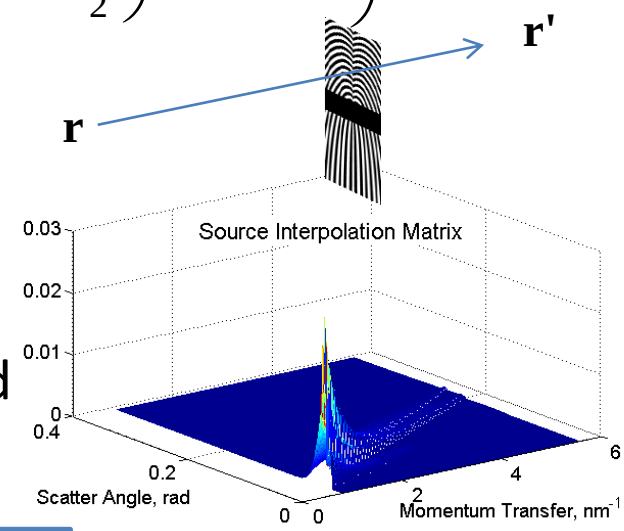
$$g(\mathbf{r}')=\int dA\int dq H(\mathbf{r}',\mathbf{r},q)\, f(\mathbf{r},q)$$

$$H(\mathbf{r}',\mathbf{r},q)=\frac{\mathrm{C}}{dA}\left|\frac{\mathbf{n}\cdot\hat{\mathbf{s}}}{s^2}\right|T(\mathbf{r},\hat{\mathbf{s}})\left(\frac{1}{2q\sin\frac{\theta}{2}}\right)W\left(\frac{q}{2\sin\frac{\theta}{2}}\right)$$

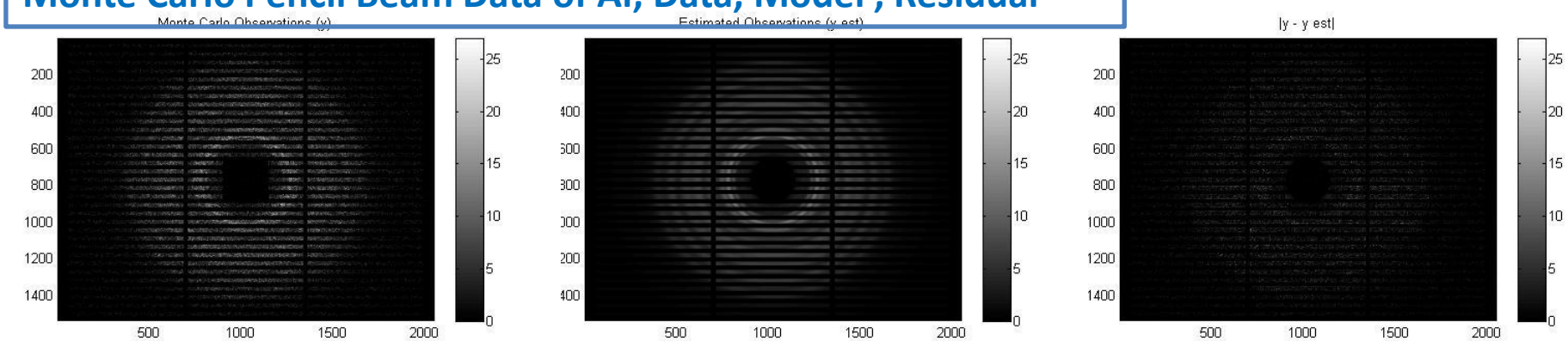
Computation: Forward Model

$$g(\mathbf{r}') = C\int d\mathbf{r}\left(\left|\frac{\mathbf{n}\cdot\hat{\mathbf{s}}}{s^2}\right| T(\mathbf{r},\hat{\mathbf{s}})\int dq\left(\frac{1}{2q\sin\frac{\theta}{2}}\right) W\left(\frac{q}{2\sin\frac{\theta}{2}}\right) f(\mathbf{r},q)\right)$$

- Detector response = Integrate object points × geometry factor × mask × integrate object momenta at scatter angle
- There exist opportunities to exploit symmetry
- Efficient computations have been implemented
- Backward model is the adjoint operator



Monte Carlo Pencil Beam Data of Al; Data, Model , Residual



Log-likelihood for Poisson Data

$$g(\mathbf{r}') = \int dA \int dq H(\mathbf{r}', \mathbf{r}, q) f(\mathbf{r}, q) \rightarrow \mathbf{g} = \mathbf{Hf}$$

$$g(m) = \sum_{i \in I} h(m,i) f(i)$$

- Forward model predicts the mean detector values
- A Poisson model is appropriate in many applications. Denote the random data by

$$y(m) \sim \text{Poisson}\left(\sum_{i \in I} h(m,i) f(i) + \mu_b(m) \right), \; m \in M$$

- The log-likelihood function for the data is

$$l(\mathbf{y} \mid \mathbf{f}) = \sum_{m \in M} y(m) \ln\left(\sum_{i \in I} h(m,i) f(i) + \mu_b(m) \right) - \left(\sum_{i \in I} h(m,i) f(i) + \mu_b(m) \right)$$

where $\mu_b(m)$ is the mean number of background counts

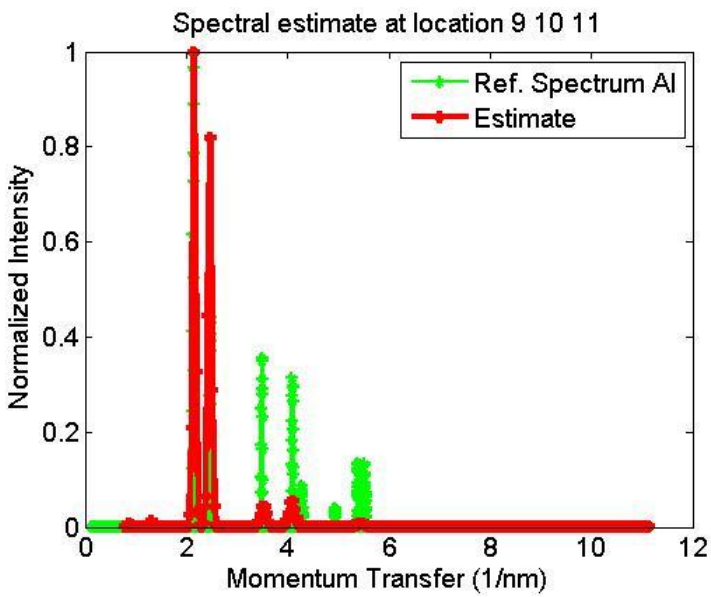
- Penalized ML estimation (also MAP); alternatively, variational Bayes (L. Carin, et al.)

$$\hat{\mathbf{f}}_{PML} = \arg\max_{\mathbf{f}} l(\mathbf{y} \mid \mathbf{f}) - \beta \varphi(\mathbf{f})$$

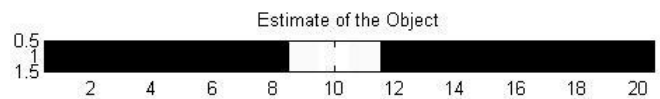
Pencil Beam Data, Forward Model and Monte Carlo

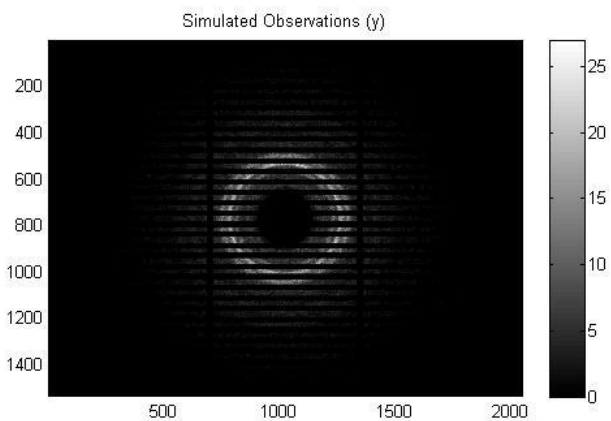
- Simulation parameters:
  - source to mask distance = 94.77 cm,
  - source to object distance = 57.78 cm,
  - source to detector distance = 109.47 cm.
- During reconstruction,
  - x resolution = 0.4 cm,
  - Number of pixels = 20,
  - momenta = 10:0.5:140,
  - downsampling factor = 1
- Simulated data: Al points at x = 9, 10, 11
- Monte Carlo data

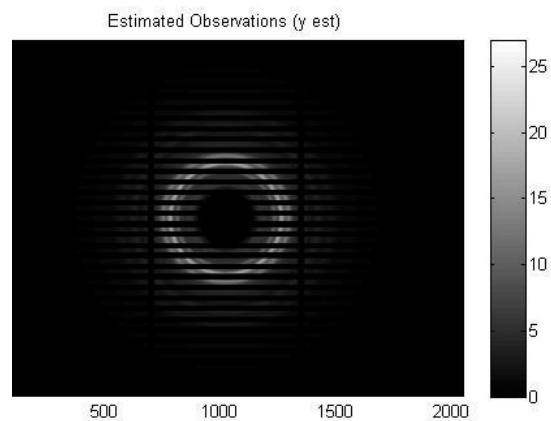
Pencil Beam Data, Forward Model After 5 Iterations

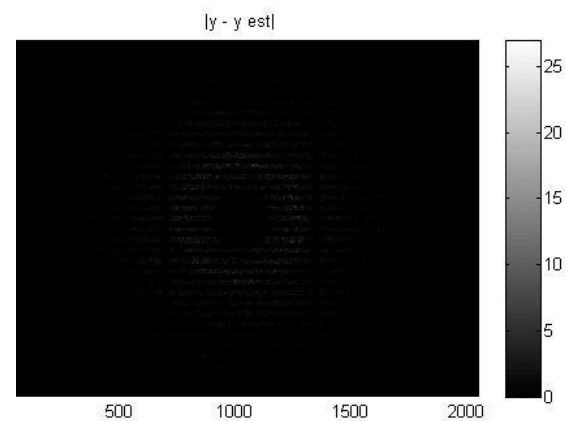


- The true Al spectrum, and the spectral estimate at location 9, 10, 11.
- Ave. of the reconstructed object over the momentum transfer coordinate.
- Simulated detector data with Poisson noise (Maximum detector value set at 50).
- Estimated detector data.
- Absolute difference between the noisy simulated and estimated data.

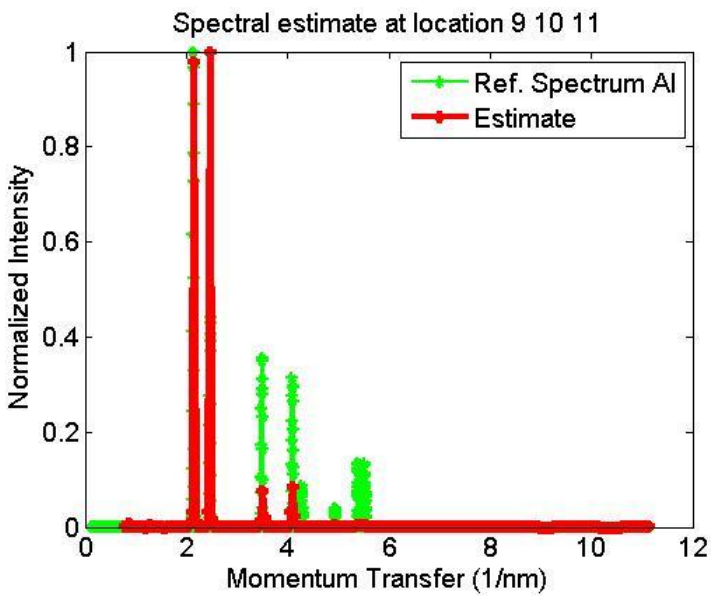




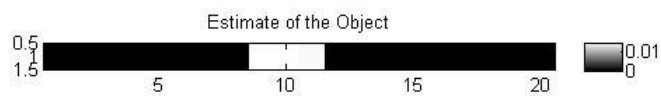


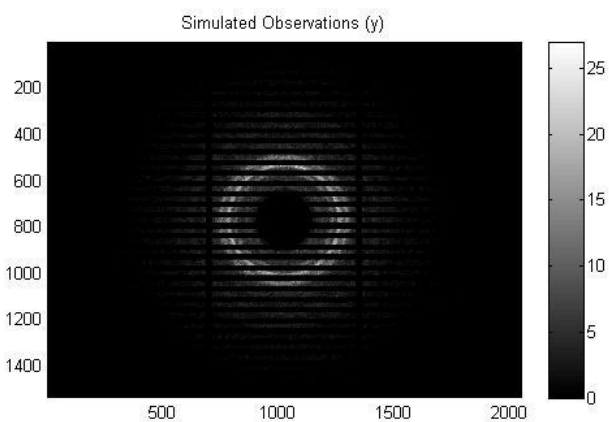


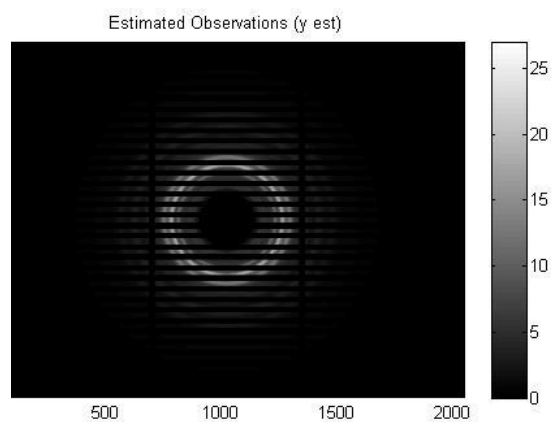
Pencil Beam Data, Forward Model After 200 Iterations

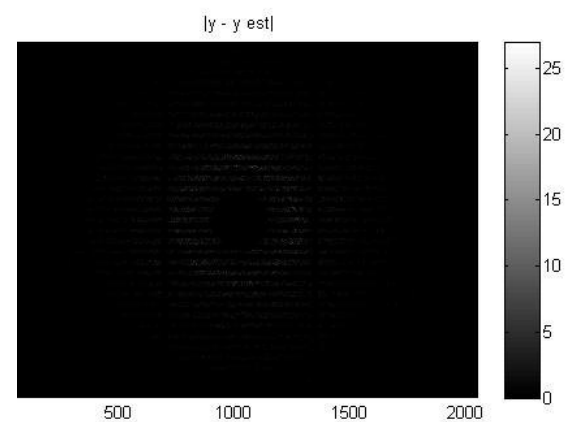


- The true Al spectrum, and the spectral estimate at location 9, 10, 11.
- Ave. of the reconstructed object over the momentum transfer coordinate.
- Simulated detector data with Poisson noise (Maximum detector value set at 50).
- Estimated detector data.
- Absolute difference between the noisy simulated and estimated data.

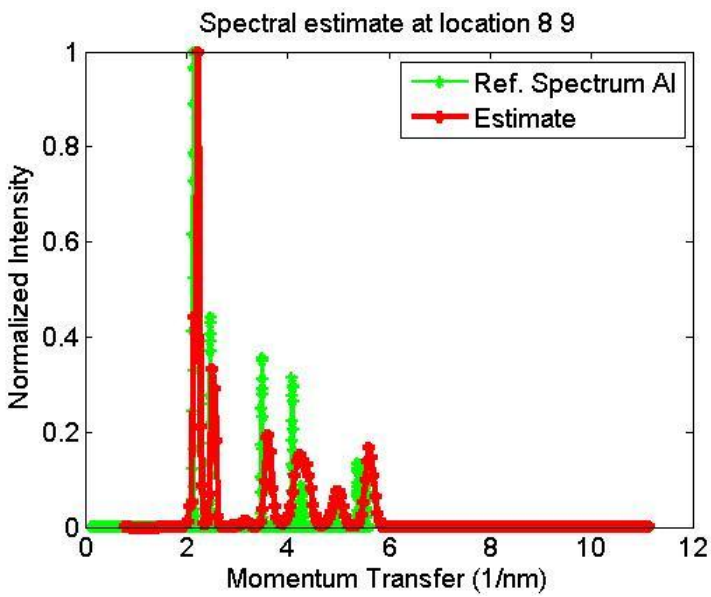




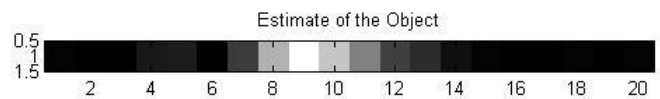


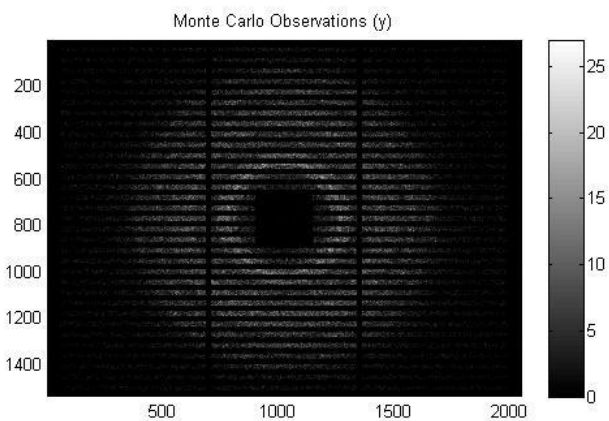


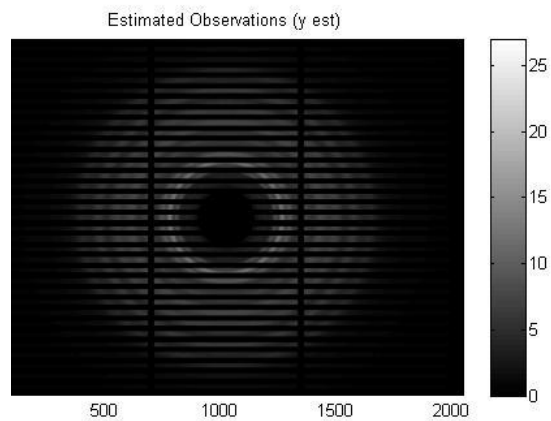
Pencil Beam Data, Monte Carlo After 5 Iterations

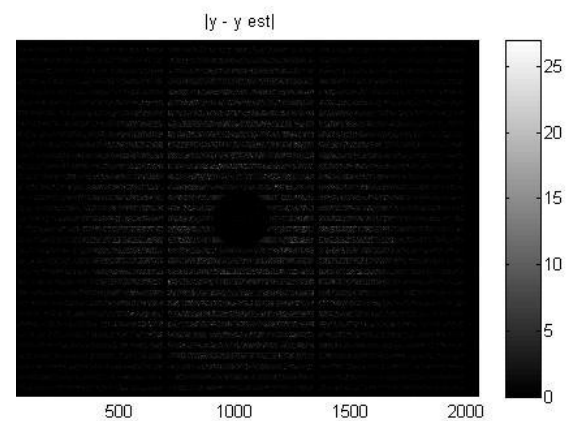


- The true Al spectrum, and the spectral estimate at location 8, 9.
- Ave. of the reconstructed object over the momentum transfer coordinate.
- The noisy Monte Carlo pencil beam data.
- Estimated detector data.
- Absolute difference between the noisy Monte Carlo and estimated data.

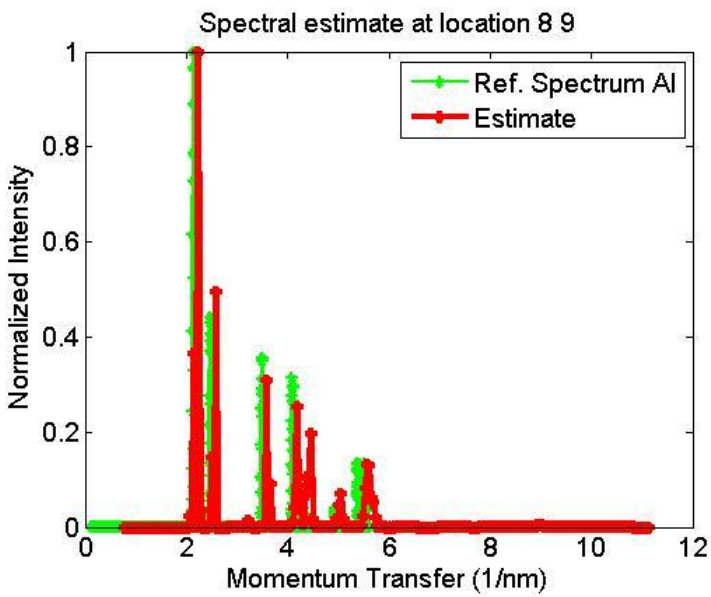




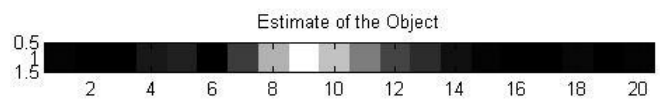


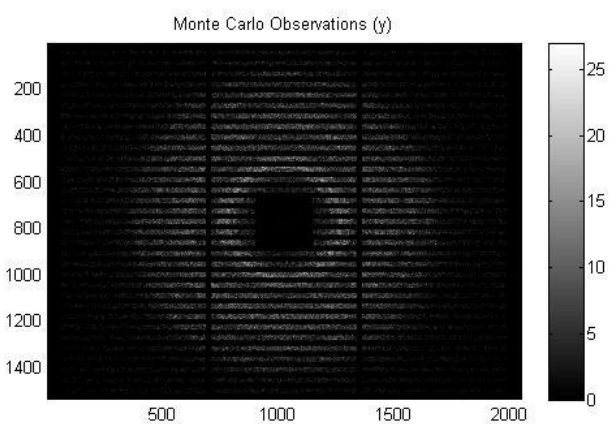


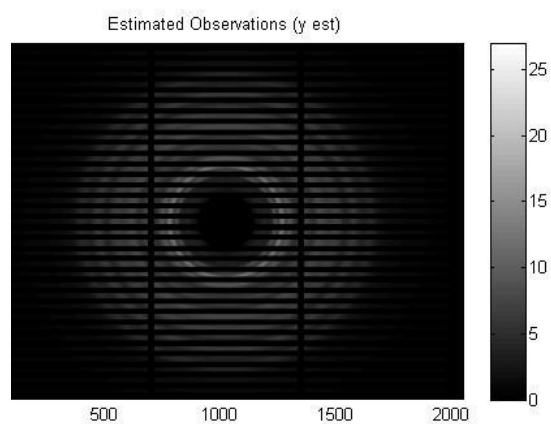
Pencil Beam Data, Monte Carlo After 200 Iterations

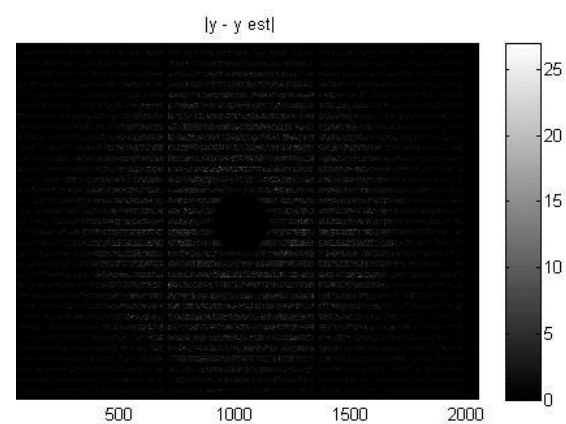


- The true Al spectrum, and the spectral estimate at location 8, 9.
- Ave. of the reconstructed object over the momentum transfer coordinate.
- The noisy Monte Carlo pencil beam data.
- Estimated detector data.
- Absolute difference between the noisy Monte Carlo and estimated data.

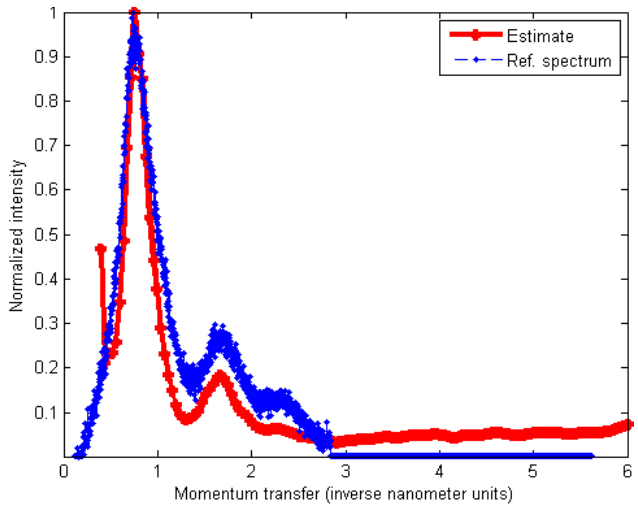




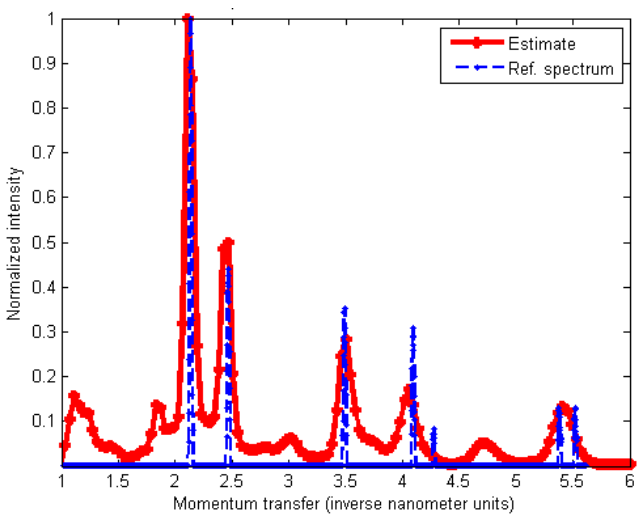




Target Signatures: Amorphous vs. Crystalline



Acrylic

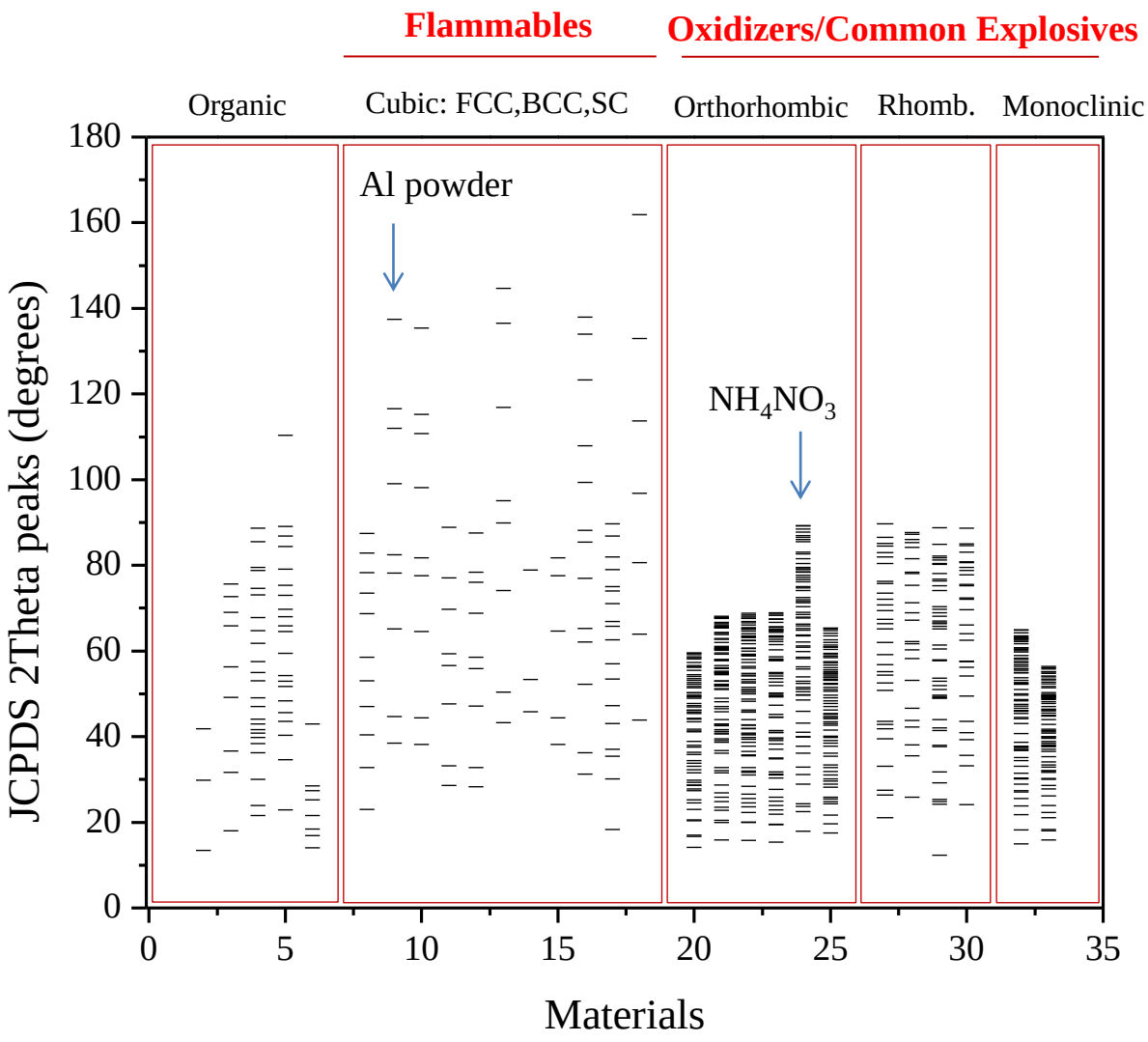


Aluminum

Crystal system: Cubic
Crystal lattice: Face-centered
(Face-centered cubic, FCC)

Diffraction spectra are dependent on crystal structure

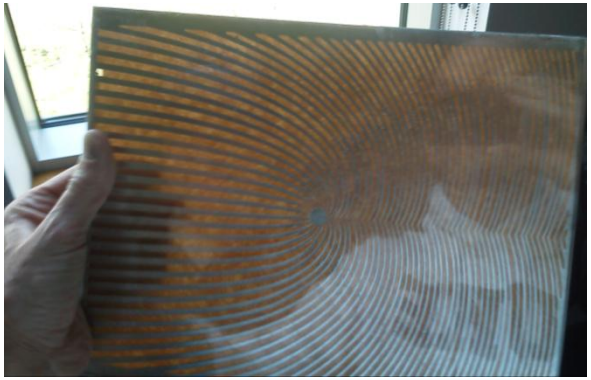
Classification based on materials crystallinity

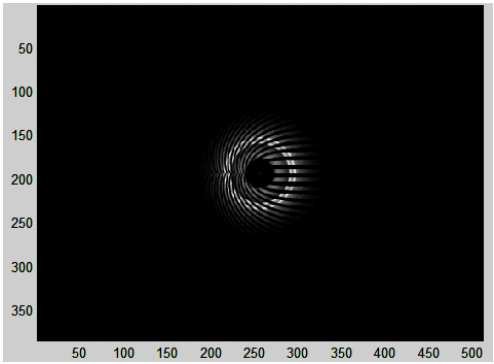


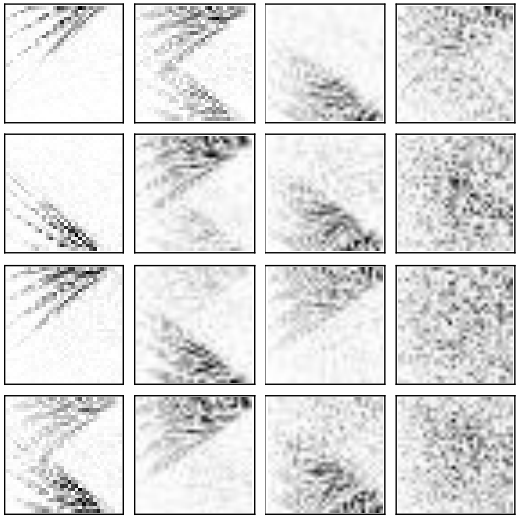
Indexing texture of diffraction pattern may aid in fine-tuning material classification

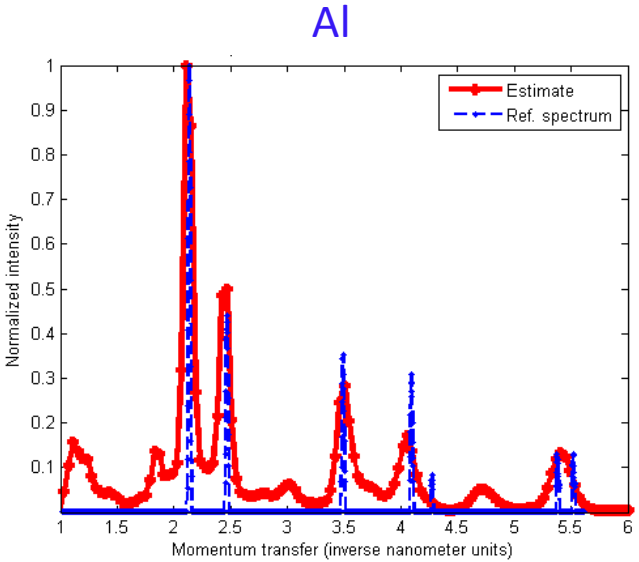
CAXSI Outline

- CAXSI System Vision
- Signature Analysis
  - Measurement space signature
    Forward models
  - Object space signature
    Reconstruction
  - Logical space signature
    SVD
- Conclusion

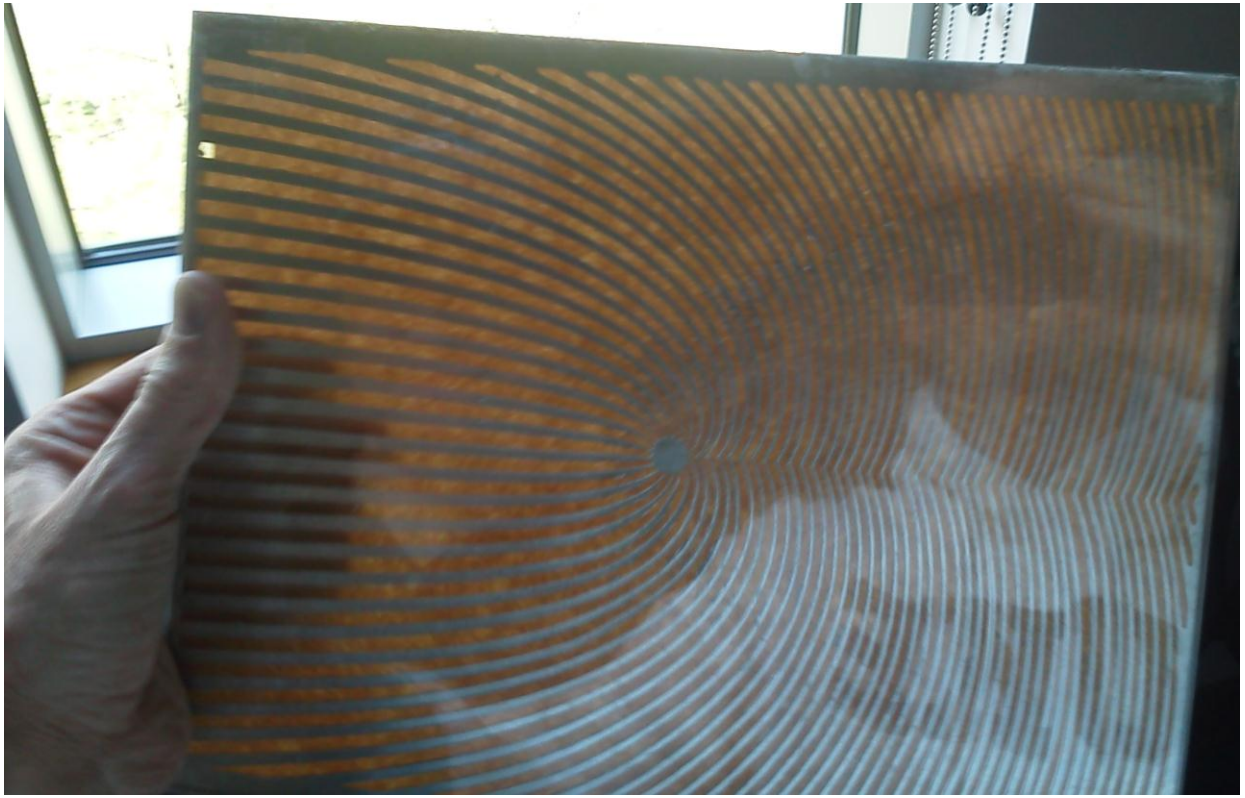




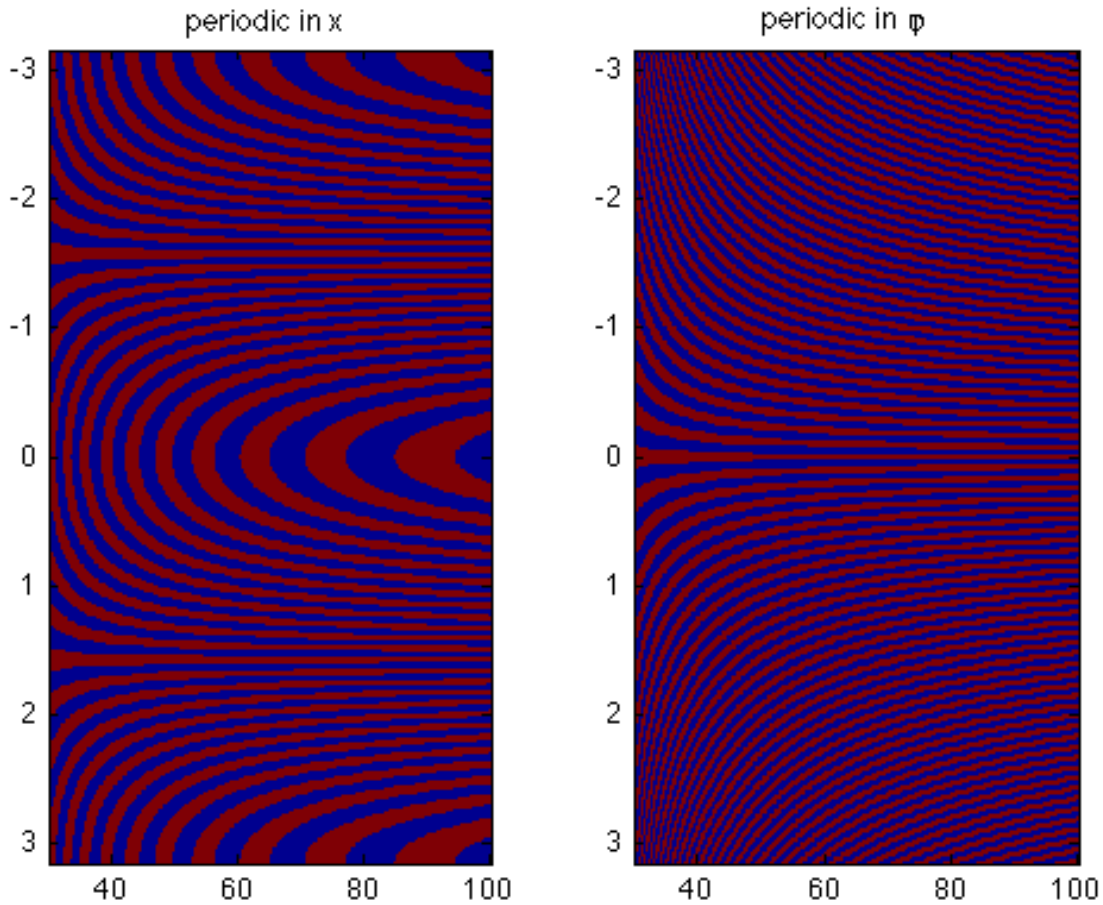




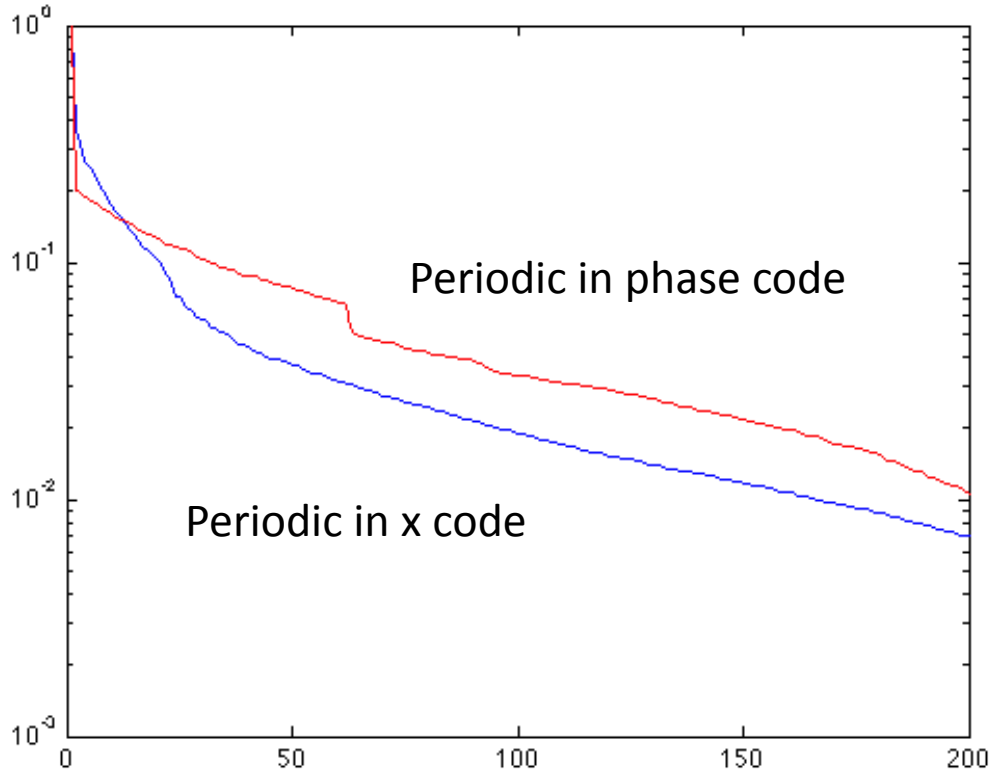
Design for Sampling Structure and Conditioning



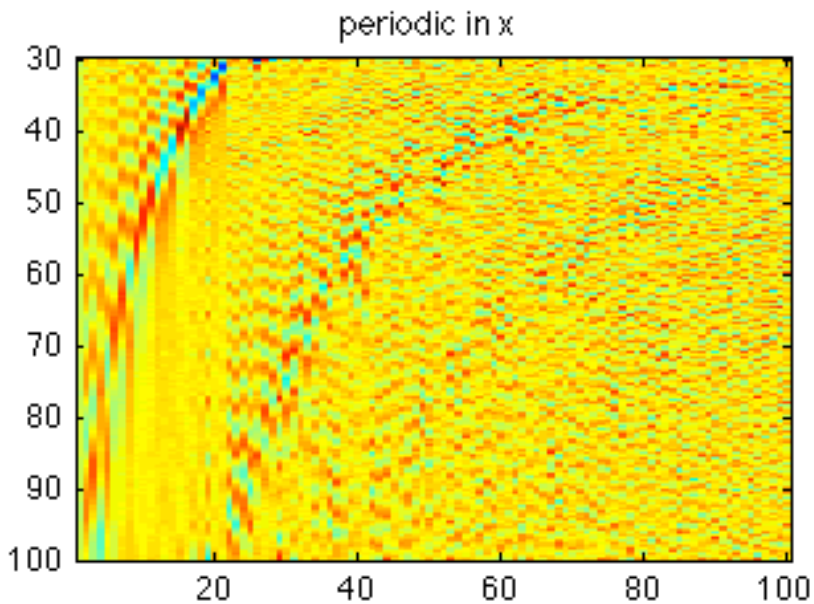
Visibility in radius and angle

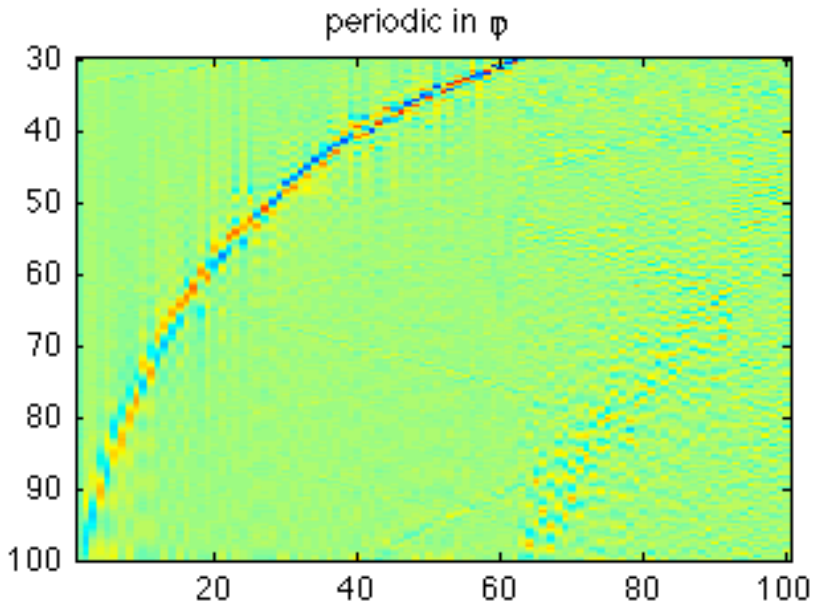


Singular Values
Pencil Beam, Coded Aperture



Singular Vectors





Singular value analysis of coded aperture x-ray scatter imaging

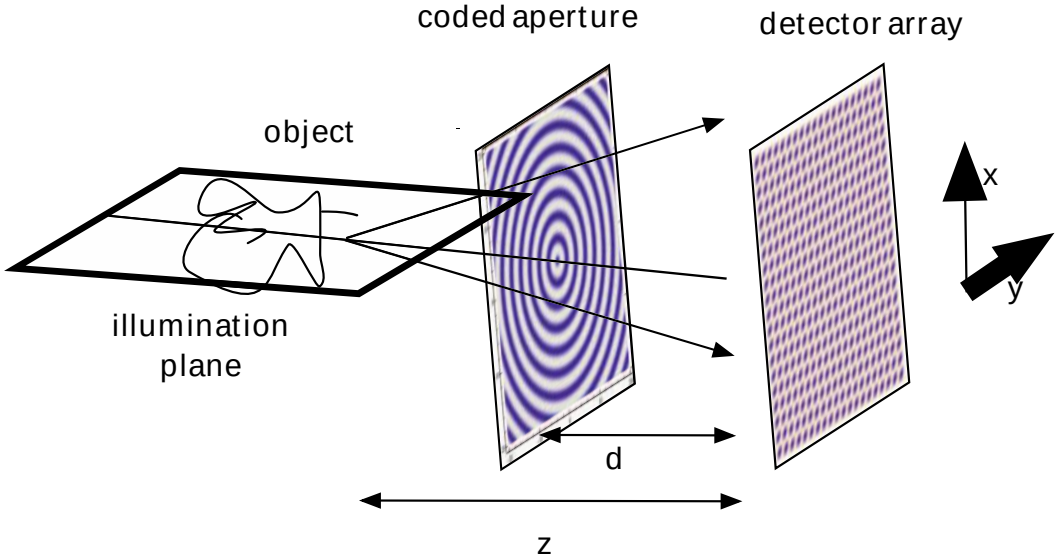
David J. Brady* and Daniel L. Marks

Fitzpatrick Institute for Photonics and Department of Electrical and Computer Engineering
Duke University, P.O. Box 90291, Durham NC 27708
*Corresponding author: dbrady@duke.edu

Compiled July 4, 2012

We examine the conditioning and singular value spectra of tomographic coded aperture scatter imagers. Scatter imaging may enable tomography of compact regions from snapshot measurements with singular values scaling favorably as compared to the Radon transform. The scaling of the singular value spectrum of the 2-D fan-beam geomery is confirmed through simulations.

OCIS codes: 110.6955, 110.7440.



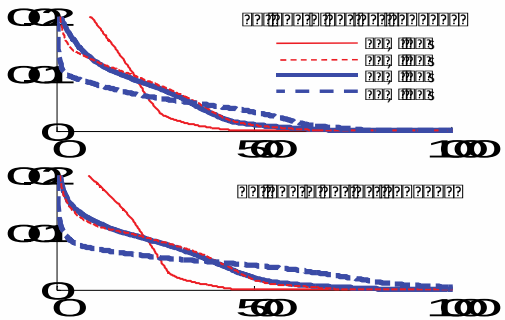
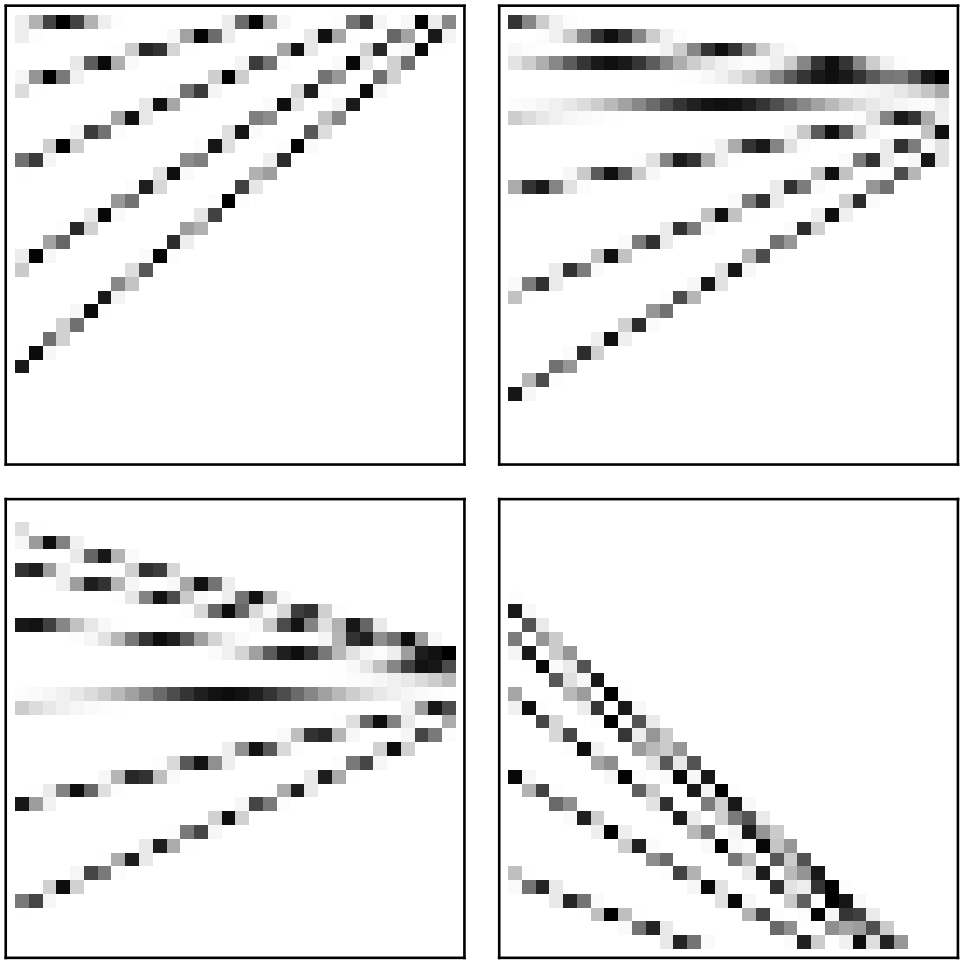


Fig. 3. Singular value spectra for (a) L = 23 length quadratic residue code and L = 47 length quadratic residue code. The four curves indicate differing number of samples measured in the H (shift code) direction, and the V (scale code) direction.

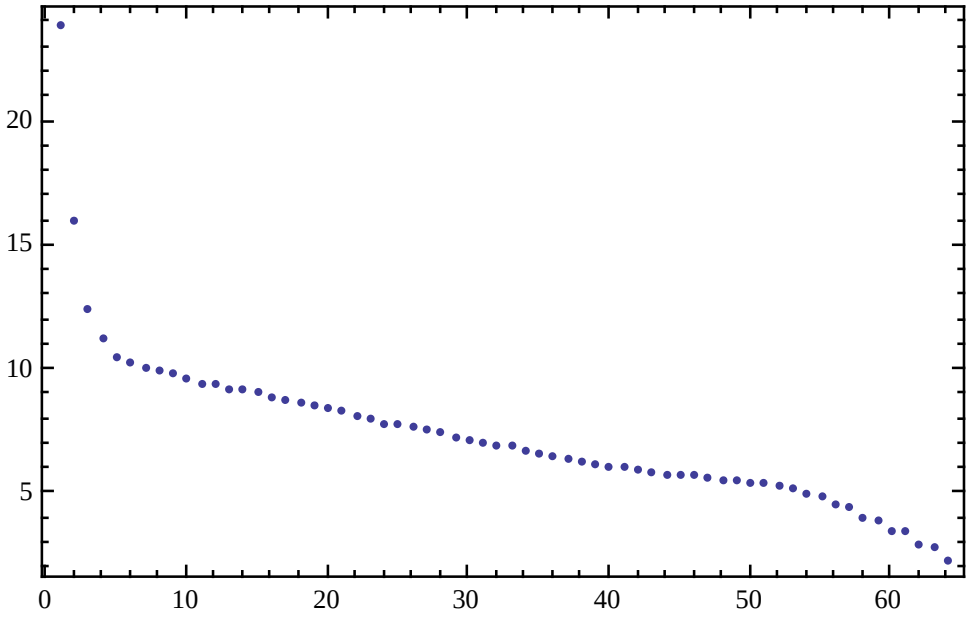
Illumination	CAXSI	Selected Volume	Radon
Pencil	$\frac{\sqrt{\Omega}}{N}$	$\frac{1}{N}$	$\frac{1}{N}$
Plane	$\frac{\Omega}{N}$	$\frac{1}{N^2}$	$\frac{1}{N}$
Volume	$\frac{\Omega}{N^{\frac{3}{2}}}$	$\frac{1}{N^3}$	$\frac{1}{N}$

Multiple Source Illumination

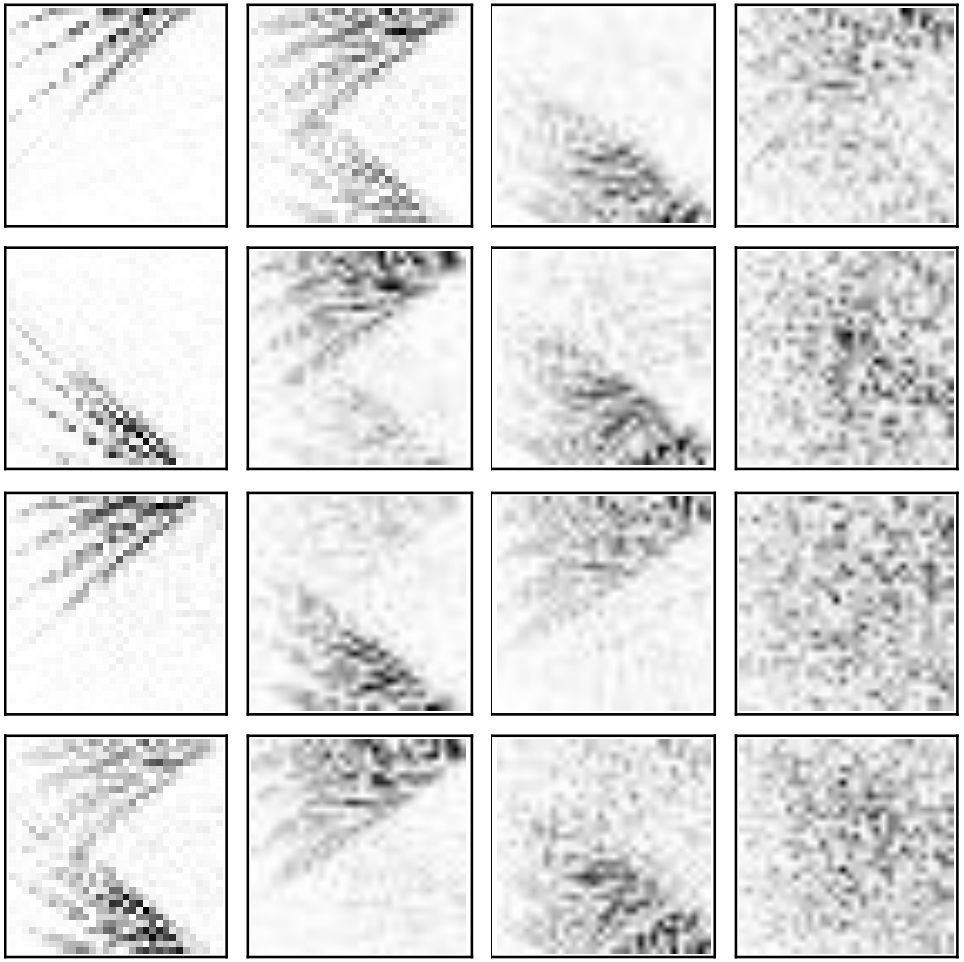


Sensor sensitivity

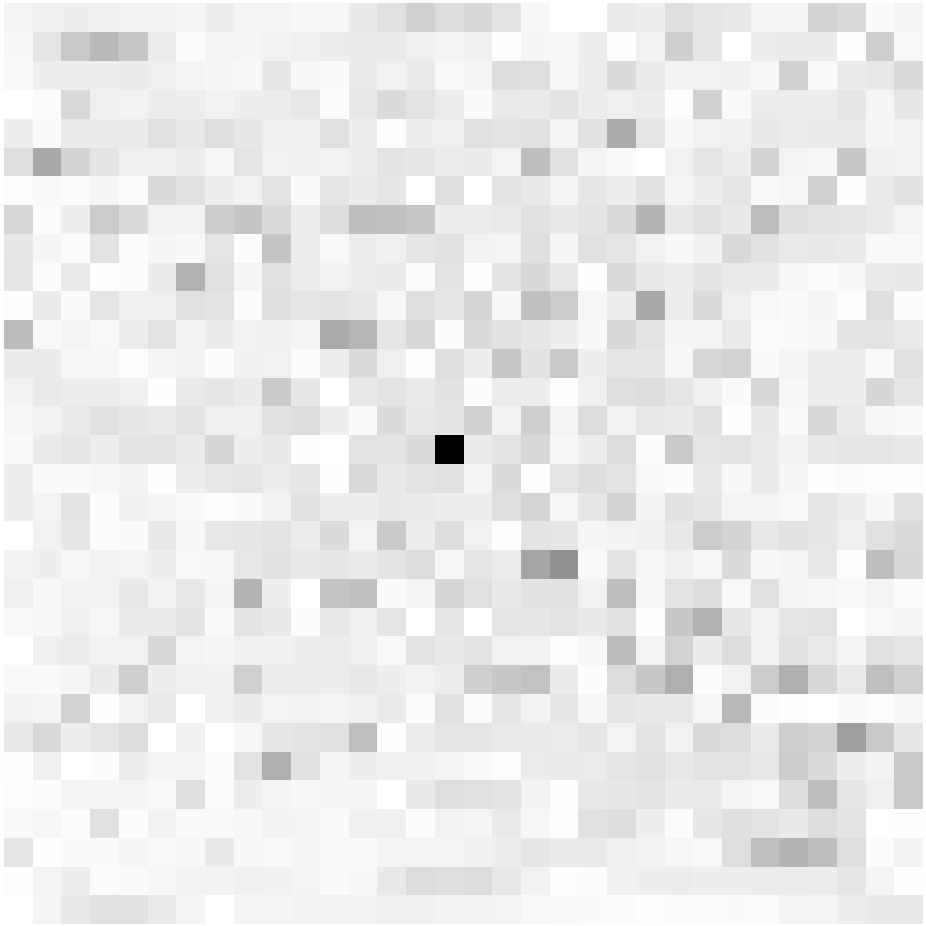
Singular Values



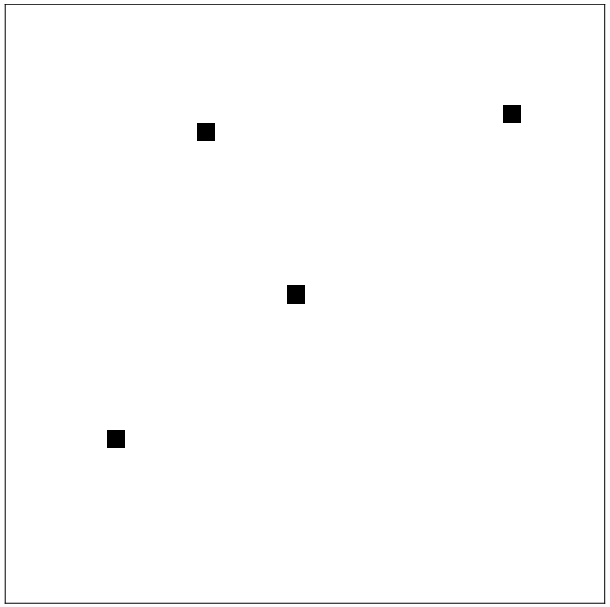
Singular Vectors



Point Target Reconstruction



Multiple Points



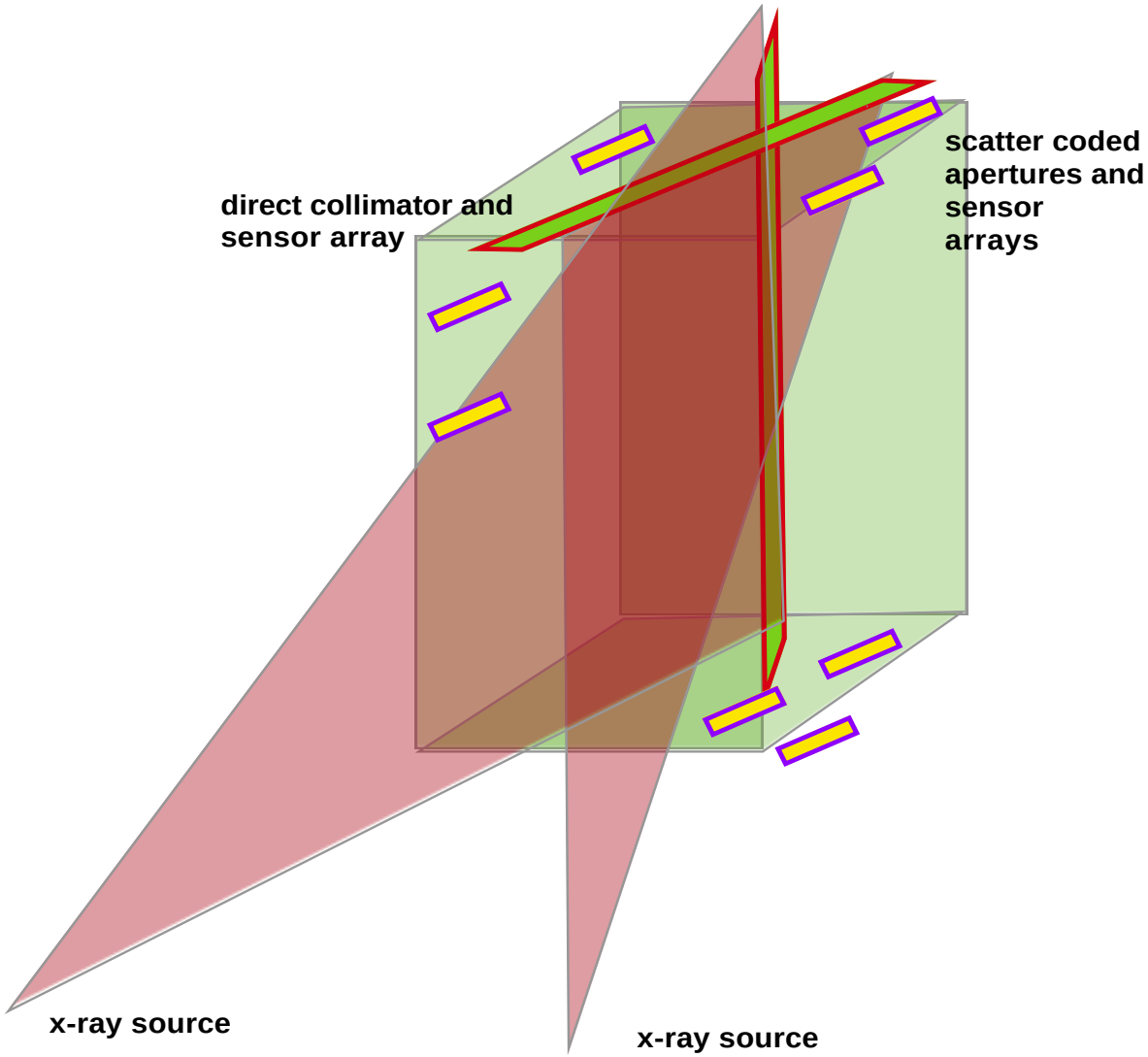
SVD and Design

- Linear response functions map generalized measurement and include detector response, source structure, object basis (dictionaries)
- Restricted isometry, source similarity etc. can be analyzed
- Linear response guides design, feeds classification engines
- System response feeds adaptive structure

Example Specifications:
Knowledge-Enhance Compressive Measurements

Tunnel geometry	60 by 40 cm 10 by 10 cm?
Source(s)	1-4 sources, multifan collimation
Beam Energy	150-160 KV
Image resolution	1.5 mm cube
Momentum resolution	0.1 nm^{-1}
System volume	3.3 (L) by 1.3 (W) by 1.3 (H) meters
Pixel size	1 mm
Number of pixels	750 for attenuation signals 5,000 scatter pixels, including 128 energy resolving pixel.

KECoM AT


direct collimator and sensor array
scatter coded apertures and sensor arrays
x-ray source
x-ray source