A Method for Simultaneous Image Reconstruction and Beam Hardening Correction

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Overview: Beam Hardening Effect in X-ray CT

❖ **Physical Acquisition Model:**

- Attenuation coefficients are energy-dependent
- X-ray energy spectrum is broad.
- The measurement model is:

$$E[y_i \mid \mu] = -\log\left(\int_R S(E)\exp\left(-\sum_j A_{i,j}\mu_j(E)\right)dE\right)$$

- y_i is measurement, $\mu_j(E)$ is the attenuation coefficient
- A is the linear forward projection matrix
- $S(E)$ is the normalized energy spectrum

❖ **Problem:**

- **Measurement is non-linear function of attenuation coefficient**

❖ **Possible Correction Approaches:**

- Hardware pre-filtering,
- Dual energy CT,
- Linearization
- Model-Based Iterative Reconstruction (MBIR), etc.

Our Approach: Simultaneous IR and BHC in MBIR Framework

❖ **A Simplified Poly-energetic X-ray Forward Model**

Idea: *Different materials can be separated according to their densities.*

- Decompose energy-dependent attenuation into two unknown basis functions

$$\mu_j(E) = x_j\left((1-b_j)r_L(E) + b_j r_H(E)\right)$$

<- Decouple location and energy

- $b_j \in \{0, 1\}$ indicator of low or high material
- $r_L(E), r_H(E)$ two underlying basis energy-dependent functions

- Substitute the decomposition into the measurement model and obtain a polynomial parameterization using two **energy-independent "material" projections**

$$E[y_i \mid x] = h(p_{L,i}, p_{H,i}) = \sum_k\sum_l \gamma_{k,l}\left(p_{L,i}\right)^k\left(p_{H,i}\right)^l \qquad p_{L,i} = \sum_j A_{i,j}x_j(1-b_j),\ p_{H,i} = \sum_j A_{i,j}x_j b_j$$

- $p_{L,i}, p_{H,i}$ two energy-independent "material" projections
- $\gamma_{k,l}$ are the coefficients of the joint correction polynomial, can show $\gamma_{0,0} = 0, \gamma_{1,0} = \gamma_{0,1} = 1$

Estimate others $\gamma_{k,l}$ online

❖ **MBIR-BHC (Simultaneous Image Reconstruction and Beam Hardening Correction)**

$$\arg\min_{x\geq 0,\gamma}\left\{\frac{1}{2}\sum_{i=1}^{M} w_i\left(y_i - \sum_k\sum_l \gamma_{k,l}(p_{L,i})^k(p_{H,i})^l\right)^2 + \text{regularization on } x\right\}$$

- Incorporate the poly-energetic X-ray forward model into MBIR objective function
- We **jointly estimate the coefficients** of the correction polynomial. **No additional spectrum information** is needed. Correction is adapted to the dataset being used.
- Material segmentation $b_j \in \{0, 1\}$ is calculated by thresholding the initial image fed into MBIR.
- Use **alternating optimization** over the image x and the polynomial coefficients $\gamma_{k,l}$

Experimental Results

❖ **Simulated Data**

- Simulated parallel-beam transmission polychromatic X-ray projections
- 720 views, 1 degree/view
- 1024 detectors, 0.24mm each
- 512x512 image, FOV = 250mm²
- Water disk phantom, with insertions of soft tissue and aluminum
- Severe streaks in FBP and generic MBIR
- Significant reduction in MBIR-BHC

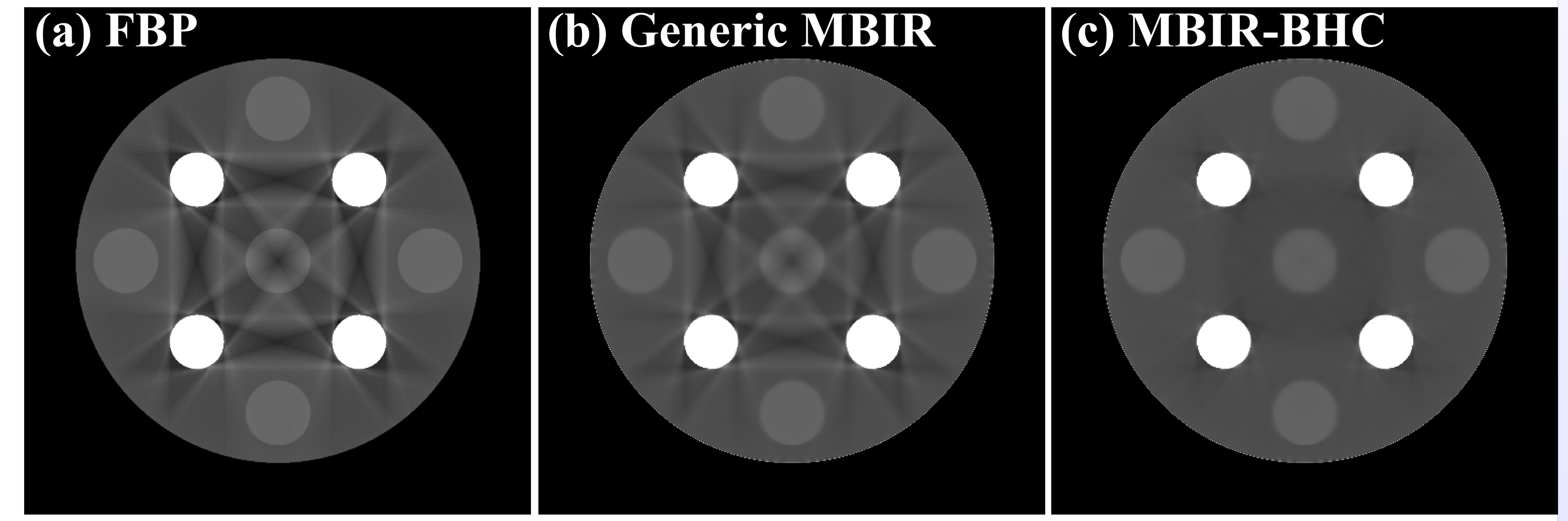


Fig. 1 Comparison of reconstructed image using the simulated dataset

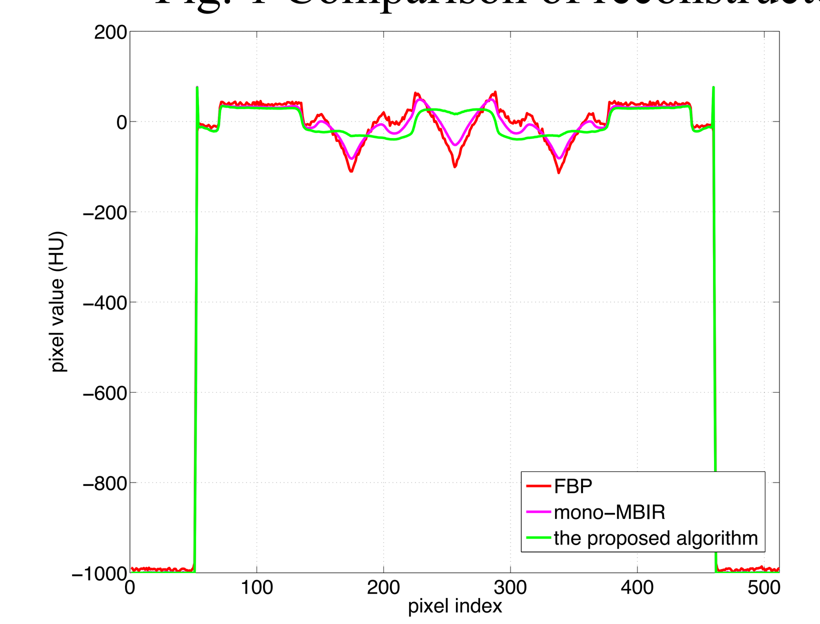
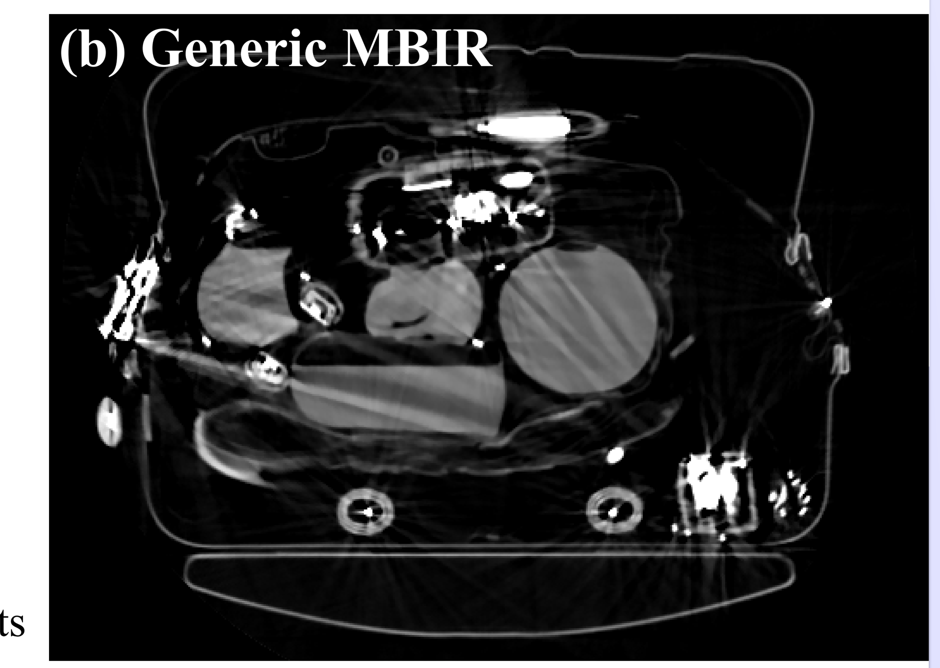


Fig. 2 Pixel profile of the simulation results

❖ **Real X-ray CT Scan**

- X-ray CT scan of an actual high clutter baggage with low and high density objects
- Resolution improvement
- Streak reduction
- Blooming artifacts reduction



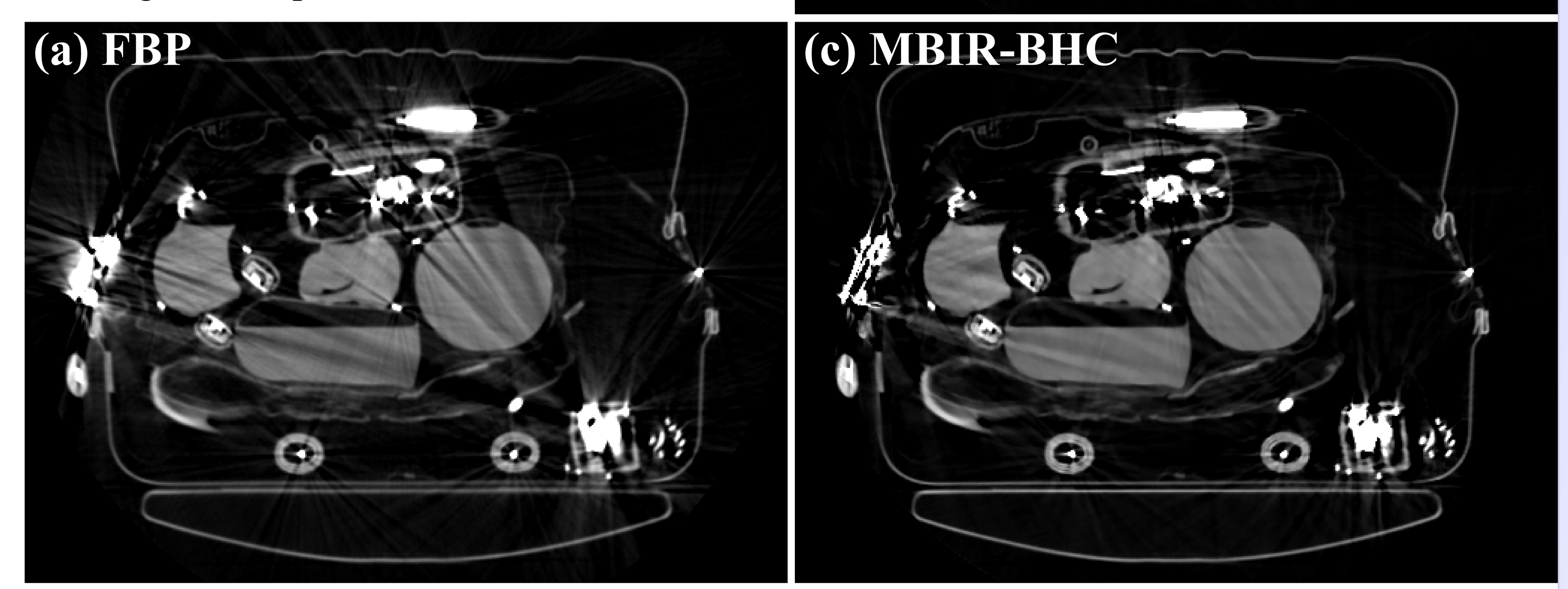


Fig. 3 Comparison of reconstructed image using the real X-ray CT data

Conclusions

- ❖ Incorporated a poly-energetic X-ray forward model using polynomial parameterization into MBIR
- ❖ Joint estimation of the image and coefficients of beam hardening correction function
- ❖ Simultaneous beam hardening correction during iterative reconstruction process
- ❖ No additional spectrum information needed, significant improvement in image quality

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