Iterative Image Reconstruction for Helical X-ray CT Baggage Scans

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This work was supported by the DHS ALERT Center of Excellence. Special thanks to Carl Crawford, John Beaty, and Michael Silevitch.
Model-Based Iterative Reconstruction

Density $x$ $\rightarrow$ Image $\hat{x}$ $\rightarrow$ Error Sinogram $(y-Ax)$

Sinogram $y$ $\rightarrow$ Fwd Model $f(x) = Ax$

Cost Function $\hat{x}_{MAP} = \arg\max_{x \geq 0} \{ \log p(y \mid x) + \log p(x) \}$

$= \arg\min_{x \geq 0} \left\{ \frac{1}{2} (y - Ax)^T D(y - Ax) + U(x) \right\}$
Scanner forward model

Path of Helical Scan

\[ y_i = \ln\left( \frac{T}{i} \right) \]  Attenuation

\[ i \] Photon count at detector

\[ T \] Air calibration scan

• Need to accurately and efficiently model the:
  • 3D forward projection geometry
  • Detector and source geometry and physics
  • Noise and distortion
Data model

- Taylor expansion of Poisson log likelihood produces

\[ \log p(y \mid x) = \frac{1}{2} (y - Ax)^t D (y - Ax) \]

- \( y_i = \log \left( \frac{I_i}{T_{i,i}} \right) \) where \( I_i \) and \( T_{i,i} \) are measured photon counts

- Matrix \( A \) is a linear projection operator

- \( D \) is a diagonal noise weighting matrix

\[ D_{ii} = \frac{1}{\text{noise variance}} = \frac{2}{I_i + \frac{2}{e}} \]

- MBIR uses information that FBP throws away!
  - Uses photon counts to estimate noise variance
  - This results in a data dependent ill-conditioned optimization problem
Multi-Core Parallelization of ICD

- **Implemented**
  - Parallel ICD on 24 core shared memory Linux machine with p-threads
  - Speedup allows for fast algorithm development

- **Performance issues**
  - Computation tends to be limited by memory/cashing speed, not computation
  - Memory must be organized as view, channel, row (slow to fast variables)
  - Allocation of slices to cores must balance computation/bandwidth load

- **Architecture of parallel algorithm**
  - Each core is responsible for updating voxels in a range of slices
  - Z-line updates:
    - A Z-line is a set of voxels along z, but at the same (x,y) position
    - Processors do ICD update along Z-lines
    - Leads to much better cash efficiency
Boundary condition and buffer slices

- For helical scan reconstructions, it is necessary to reconstruct buffer slices on both sides of the ROI
  - Buffer slices are discarded, but required for accurate reconstruction
  - With of each set of buffer slices is approximately half the width of detector array
  - Computation associate with buffer slices is overhead
Results: resolution and object discrimination

DFM

MBIR
Results: metal artifact reduction
Mixed power law data weighting

- Want to adjust the data weighting in the cost according to the suspected presence of metal in each projection measurement
- First using an initial reconstruction, \( x^{(0)} \), define a metal indicator for each projection \( i \),

\[
I_i = \begin{cases} 
1, & \text{if for some voxel } j, \text{ both } A_{ij} > 0 \text{ and } x_j^{(0)} > T \\
0, & \text{otherwise}
\end{cases}
\]

- Mixed data weighting: 

\[
D_{ii} = I_i \left( \frac{i}{T,i} \right) + (1 - I_i) \left( \frac{i}{T,i} \right)^{0.5}
\]

where \( i \) is the target scan count and \( T,i \) is the air scan count
Results: power law data weighting

DFM

MBIR

\[ D_{ii} = \begin{cases} i \\ \frac{1}{2} \end{cases} \]

MBIR

\[ D_{ii} = \text{mixed} \{ i, i^{0.5} \} \]
Results: object discrimination

DFM

MBIR
Results: metal artifact reduction

DFM

MBIR
Results: artifact reduction

DFM

MBIR

Iterative Reconstruction for Helical CT baggage scans

23 October, 2013
Detector afterglow correction

before

after
Fan angle offset correction

DFM w/out correction

MBIR with correction
Industry/University Collaboration

- My background:
  - 12 year GE relationship: *Veo* and 3T MRI
  - 20 years HP relationship: Technology in millions of printers
  - Signal Processing: Applied math, algorithms, physical models

- The opportunity:
  - Technology transfer from university to large company
  - Build on company’s infrastructure
  - Provide university an efficient path to impact

- The obstacles:
  - Trust, IP, information sharing, risk
  - Understanding need to make money
  - Understanding need to publish and educate

- The keys to success:
  - Industry researcher who takes ownership
  - University researcher committed to success
  - Technology that will differentiate industry in the marketplace
Summary

- MBIR offers great potential in baggage screening
  - Improved resolution
  - Reduced artifacts
  - Increased design flexibility
- Model accuracy is important
- Computation remains a challenge

- Key’s to success in industry university partnership:
  - Trust
  - Committed team of researchers on both sides
  - Tight integration of research with clear goals
MBIR/Veo Publications and Patents

• Some key publications:

• Issued patents:
Distance-Driven (DD) forward projector

- CT forward projection is modeled by a linear matrix operation.

\[
\begin{bmatrix}
y
\end{bmatrix}
= \begin{bmatrix}
A
\end{bmatrix}
\begin{bmatrix}
x
\end{bmatrix}
\]

- The j-th column of A corresponds to projection of voxel j.

- In DD model, each voxel is flattened along the dimensions parallel to detector face.

- Each column entry is calculated as a product of XY-plane projection \(B_{i,j}\), and Z-direction adjustment factor \(C_{i,j}\) for i-th detector element.

\[
A_{i,j} = B_{i,j} \cdot C_{i,j}
\]
The forward projection matrix $A$ is calculated as

$$A_{i,j} = B_{i,j} \cdot C_{i,j}$$

$$B_{i,j} = \frac{D_{xy}}{\cos(\phi) \cdot \min(W_c, |c|)}$$

$$C_{i,j} = \frac{1}{d_r \cos(\phi) \cdot \min(W_r, d_r)}$$

$\delta_c, \delta_r$ : Detector width in channel and row directions

$W_c, W_r$ : Voxel projection width in channel and row directions

: Ray angle in XY-plane and Z-direction

: Offset from detector element center

: Voxel size
Poisson noise model

- Use a 2\textsuperscript{nd} order Taylor series expansion of true log likelihood

$$\log p(y | x) = \sum_{i=1}^{M} \left( \exp \left( A_{i,*} x \right) + \frac{1}{2} (y - Ax)^t D (y - Ax) + c \right)$$

where

- $A_{i,*} = i^{th}$ row of $A$
- $y_i = \log \left( \frac{0,i}{i} \right)$
- $D_{i,i} = i$

- $A$ - forward system matrix
- $D$ - diagonal weighting matrix
Iterative Coordinate Descent (ICD)

- Iteratively match each pixel (i.e. each column of $A$)
- Select each pixel to minimize total cost

$$x_j \leftarrow \arg\min_{x_j} \left\{ \frac{1}{2} \|y - Ax\|^2 + U(x) \right\}$$

**Issues:**
- Efficient update by using sinogram error state
- High spatial frequencies converge first
- Benefits from good initial condition

Why ICD?

• Advantages:
  • Fast convergence at high spatial frequencies
  • Can be initialized with FBP
  • Sequence of 1D updates provides flexibility
  • Easy to enforce positivity constraints
  • Robust to non-idealities

• Disadvantages
  • Poor low frequency convergence
  • Irregular memory access
RMSE Convergence Plots for NH-ICD

- NH-ICD
  - Reduces transients at early stage allowing faster convergence
  - Interleaving in early iterations further improves convergence speed

Image prior model

\[ U(x) = \frac{1}{p} \sum_{j,k \in C} (x_j - x_k)^p \]

- 3D regularization using 26 neighbors
- Design to:
  - Preserve high contrast edges
  - Enhance low contrast sensitivity

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where \( p = 1.2 \)

with \( p = 1.2 \) and \( q = 2 \)

Prior: Q-Generalized Gaussian MRF

• Define neighboring pixel difference \( \Delta = x_s - x_r \). The q-GGMRF prior is defined as

\[
p(x) = \frac{1}{z} \exp \left\{ - \frac{1}{q \sigma^q_x} \sum_{\{s,r\} \in C} g_{s,r} \rho(\Delta) \right\}
\]

where \( \rho(\Delta) = \frac{|\Delta|^p}{1 + \left| \frac{\Delta}{c} \right|^{p-q}} \)

• Controls both low and high-contrast behavior

• Parameter \( c \) is a soft transition point such that

\[
( \quad ) \approx \begin{cases} 
| \quad |^p \text{ for } | \quad | << c \\
| \quad |^q \text{ for } | \quad | >> c 
\end{cases}
\]

• Gaussian MRF (GMRF) prior is the special case where \( p = q = 2 \), i.e. \( ( \quad ) = 2^2 \)
Non-Homogeneous ICD (NHICD)

- Objective: find good correlation between update map and true RMS error at different stages of

Top 5% pixels with largest update values at iteration 1

Top 5% pixels with largest RMS error at iteration 1
Model-Based Iterative Reconstruction

- Our framework is the maximum a posteriori (MAP) estimate

\[
\hat{x}_{MAP} = \arg \max_{x \geq 0} \{ \log p(y \mid x) + \log p(x) \}
\]

- Vector \( y \) is the projection measurements, and \( x \) is the image

- MBIR is used in GE Healthcare’s Veo product which is sold in US and European markets since 11/2011

- We are working with Morpho Detection to investigate the use of MBIR in an EDS system for aviation security
Evaluation for EDS performance

- Evaluated qualitative impact of model-based reconstructions on proprietary automatic threat detection (ATD) algorithms
  - Improved segmentation
  - Improved object identification/classification
  - Improved separation of adjoining objects
  - Reduction in false alarms

- In addition, the improvements in reconstruction quality provide for better operator experience

- Reduced cost of additional detection