Estimation and Detection Information Tradeoff for X-ray System Optimization

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So What? Who cares?

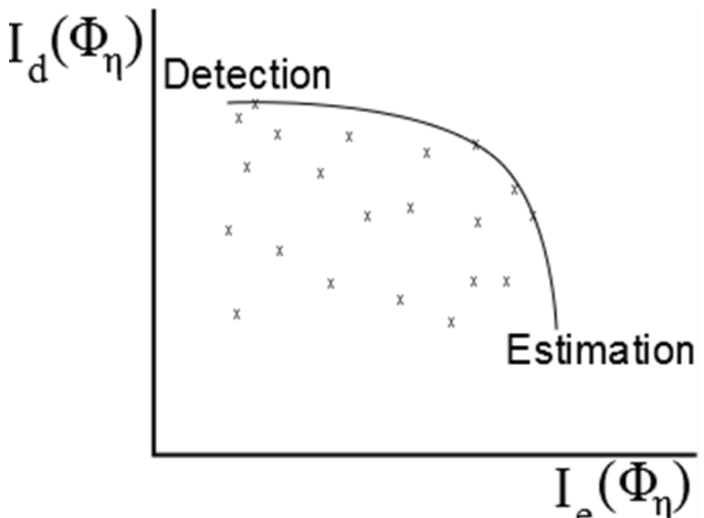
Problem statement:

We consider the case where the parameter vector θ to be estimated is associated with objects from both classes and that what distinguishes the two classes is the prior distribution on this parameter.

Why you should care

- Provides method of measuring the performance of Imaging systems in the joint task case.
- Flexibility allows application of preferred detection and estimation metrics.
- Stresses the tradeoff between detection and estimation performance.
- Allows consideration of environment to calibrate scanner appropriately.
- Based in optimization of the average cost function.

$$\tilde{C} = P_0 \tilde{C}_{10} \Pr(FP) + P_1 \tilde{C}_{01} \Pr(FN) + P_0 \left\langle C\left(\hat{\theta}(g), \theta\right)\right\rangle_{TN} + P_1 \left\langle C\left(\hat{\theta}(g), \theta\right)\right\rangle_{TP}$$



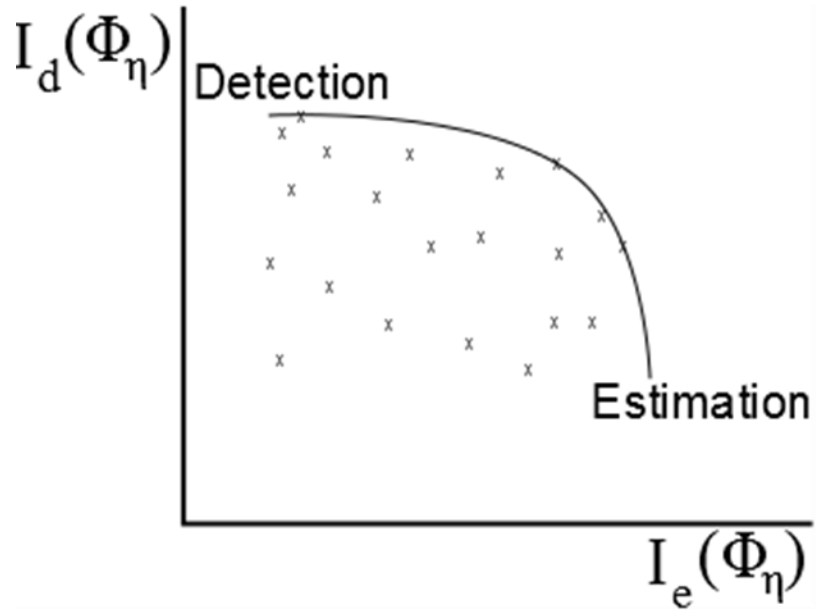
Why you should be skeptical

- Does not provide a scalar solution.
- To get definitive solution must apply costs matrix.

How does it work?

EDIT Curve[1]:

Average Cost: $C = P_0[C_{10}\,\Pr(FP) +$

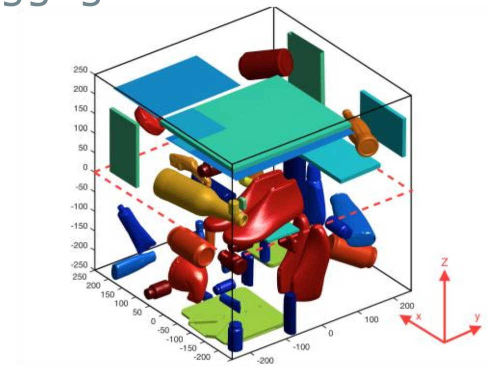


Two methods of calculation

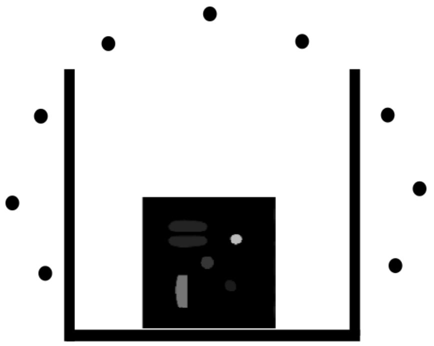
- Can calculate this directly by finding the maximizing parameter vector ϕ.
- Could create a number of systems and treat the convex hull of these point as an approximated EDIT.

So What? Have you done anything with EDIT?

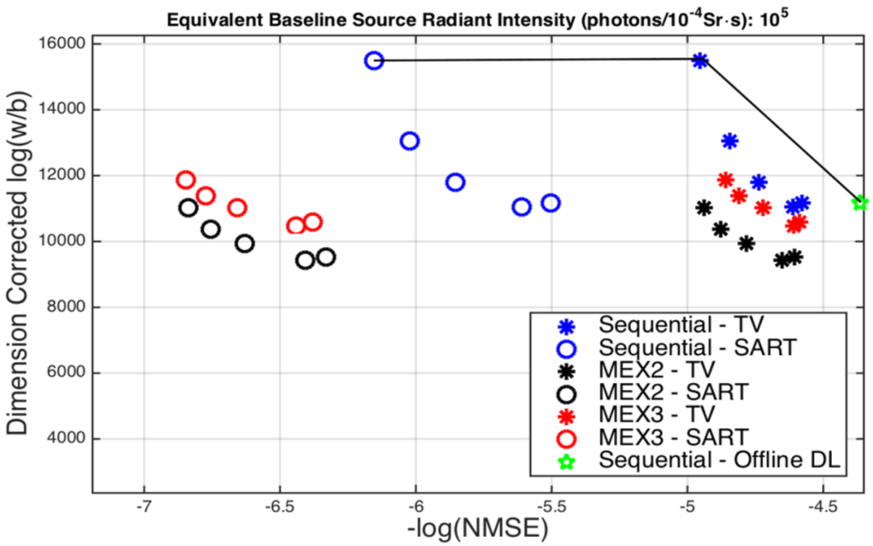
- Used a series of stochastically generated bags to test the performance of a fixed geometry baggage scanner.



Stochastic Bag



Fixed geometry slice scans



Example of curve generated.

- Method used whereby the convex hull of the data generated sets the EDIT curve

Unique Conclusions

- At low SNR Sequential scanning outperforms multiplexed scanning.
- Spectral binning reduces estimation performance but increases detection performance at these SNR levels.

Is that all? Are you working on anything now?

We have started using real scanner systems to generate data!

What is being done:

Using a fan beam CT scanner built at Duke university and scanning test bags we have been analyzing the effect of spectral systems with different angular under sampling and different reconstruction methods.

What are the potential outcomes:

- Confirming or refuting simulated data findings.
- Finding an optimal number of spectral bins.
- Finding the best reconstruction techniques for angularly under sampled scans.

Current work being headed by Sagar Mandava and Dr. Ali Bilgin with oversight from Dr. Amit Ashok.

That seemed awfully vague, how did you do your original project exactly?

The pieces that make up DHS project:

- **Stochastic Bag generation**
- Forward Model (Using ray trace methods)
- Detection Information (CSMI's log w/b metric)
- Reconstruction Methods (SART,TV,Offline DL)
- Estimation Information (Normalized MSE)
- EDIT Curve calculation
- RESULTS
- Conclusion

Stochastic Bag generation

The Stochastic bag generator:

- Collection of objects defined as *.stl files (faces and vertices)
- Objects selected randomly based on likelihood of appearing in bag and placed in random locations.
- Objects continually placed in bag until failure to fit new object without overlap or max limit reached.
- For threat bags last object is changed into either a threat object (change in shape) or a threat material (change in material properties.
- Data for bag geometry stored in JSON files to allow easy access in several languages. (MATLAB,C++, and python are used)

For this experiment we used:

- 100 Threat (shape based bags)
- 100 Non-threat bags
- Bags were $500mm^3$ in volume

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How do you run your virtual bags through your virtual machine?

- The ray trace based method defined source location and cavity geometry. The source strength and scan time was then used to generate a number of rays in random direction within the source emission output angle.
- A GPU based algorithm then propagated each ray from its source to the first object of contact. At this point scattering and absorption effects could be calculated
- Once a path from source to detector was traced the total absorption effects of materials along the path were then used to adjust source ray strength.

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What detection metric did you use for EDIT?

CS Mutual information used for detection information[4]:

CSMI measures the divergence between 2 pdfs. equivalent to $\log\left(\frac{w}{b}\right)$.

- $I_{CS}(\vec{g},C) = -\frac{1}{2} log \frac{\left[\sum_C \int [p(g|C)p(C)][p(g)p(C)]dg\right]^2}{\sum_c \int p^2(g|C)p^2(C)dg \cdot \sum_C \int p^2(g)p^2(C)dg} = \frac{1}{2}\left(\log(2) + \log\left(\frac{w}{w+b}\right)\right)$

This expression can then be simplified further using a modified Bessel function. The end result is a quick to calculate metric that for our systems was shown to correlate with Pe (Probability of error). See reference 4 for more information.

Remember EDIT does not require the use of these metrics. It would be best to employ the same detection algorithm you plan to use in the field for analysis.

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You had to of created images at some point.

Our detection method is accomplished entirely in the image space of the system. This means that we did not need to reconstruct in order to detect. For estimation though, not only must a gold standard be created but a reconstruction of data must also be performed.

Gold standard:

- To create a gold standard a voxelizer program was developed which took the parsed data of a json file and generated a 3D matrix containing the absorption values at each location.

Reconstruction methods:

We compared 3 different reconstruction techniques.

SART- Simultaneous Algebraic Reconstruction Technique

$$x^{k+1} = x^k + \lambda_k \mathrm{A}^T(y - Ax)$$

Iterative based reconstruction technique.

TV- Total variation technique

$$\hat{x} = \underset{x}{\operatorname{argmin}} \|y - Ax\|_2 + \lambda TV(x)$$

Offline DL- Dictionary Learning

$$\hat{x} = \underset{x}{\operatorname{argmin}} \|y - Ax\|_2 + \lambda TV(x) \;\; s.t.$$

$$\|DA - Px\| < \sigma, \|\alpha_i\|_0 < \tau$$

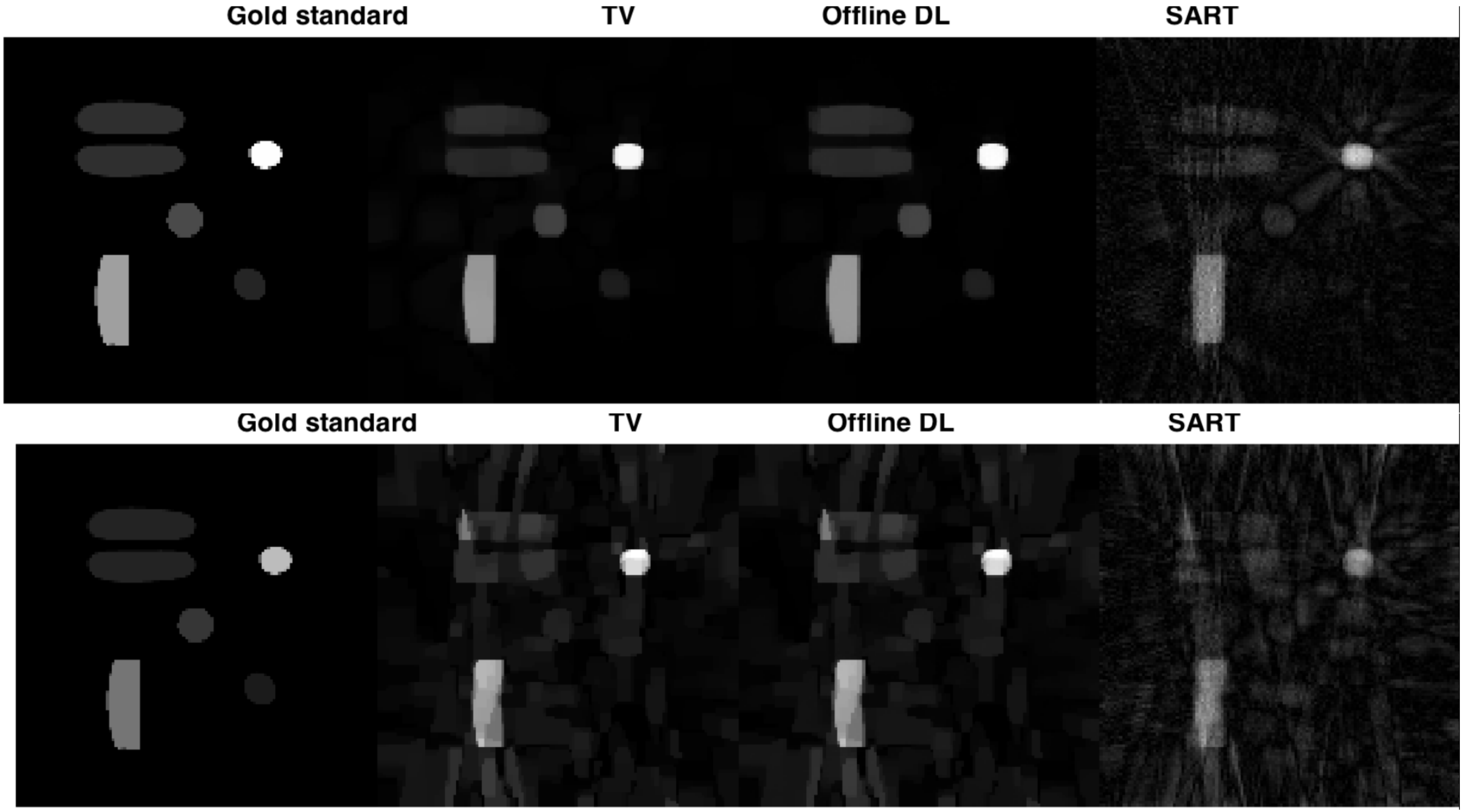
Equivalently iterate between:

$$\hat{x} = \underset{x}{\operatorname{argmin}} \|y - Ax\|_2 + \lambda TV(x)$$

$$A_{n+1} = OMP(A_n, x_n, \tau), \qquad x_{n+1} = P^{-1}(D, A_{n+1})$$

What do these reconstructions look like.

Reconstructions are slice by slice.



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NMSE

- *Normalized mean square error. Allows for unitless MSE we can then take the log of to compare with our detection metric.*

$$NMSE = \frac{1}{N}\sum_{i}\frac{(P_i - M_i)^2}{\bar{P}\bar{M}}$$

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EDIT Curve[1]:

Find:

$$\phi_\eta = \underset{\phi}{\operatorname{argmax}}\left[\tilde{C}_\eta(\phi)\right]$$

Where:

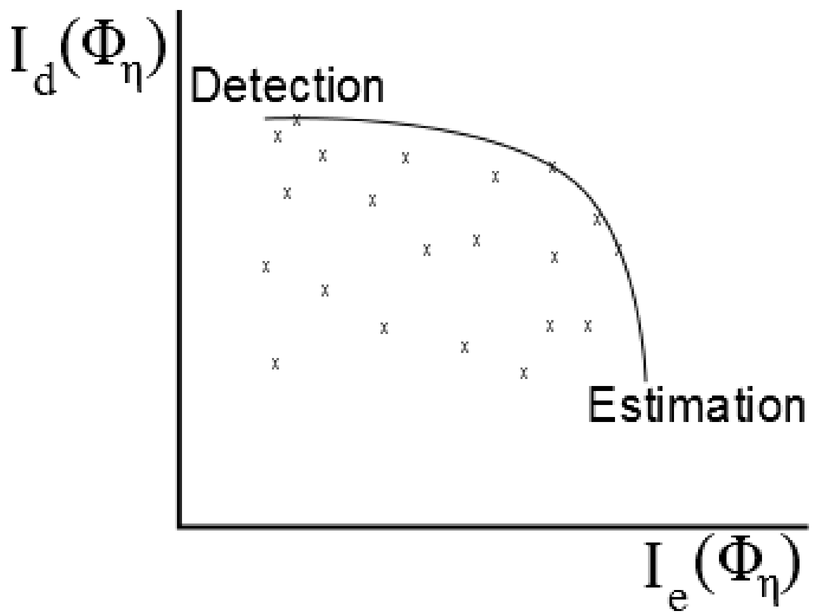
$$\tilde{C}_\eta(\phi) = I_d(\phi) + \eta I_e(\phi)$$

Plot:

$$\left(I_e(\phi_\eta), I_d(\phi_\eta)\right)$$

EDIT

- Method 1:
- Can calculate this directly by finding the maximizing parameter vector ϕ.



- Method 2:
- Could create a number of systems and treat the convex hull of these point as an approximated EDIT.

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- **RESULTS**
- Conclusion

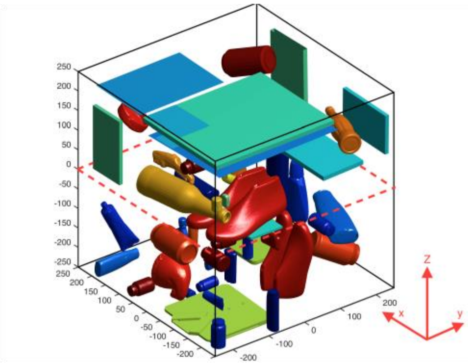
Stochastic Bag

Stochastic bag generator[1] used to create randomly filled 3D luggage objects[2].

100 threat bags

100 non-threat bags

$500mm^3$ bag volume



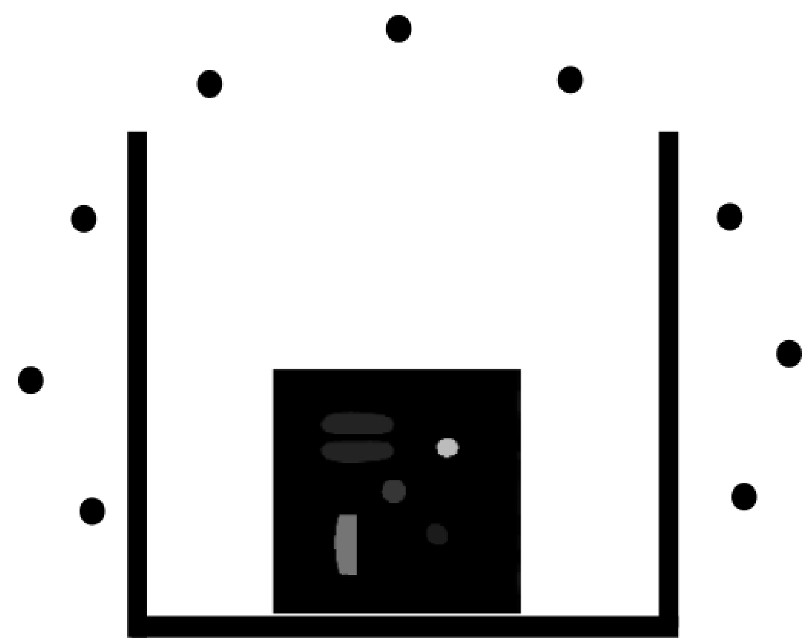
Forward Model

GPU-based ray-tracing method[3].

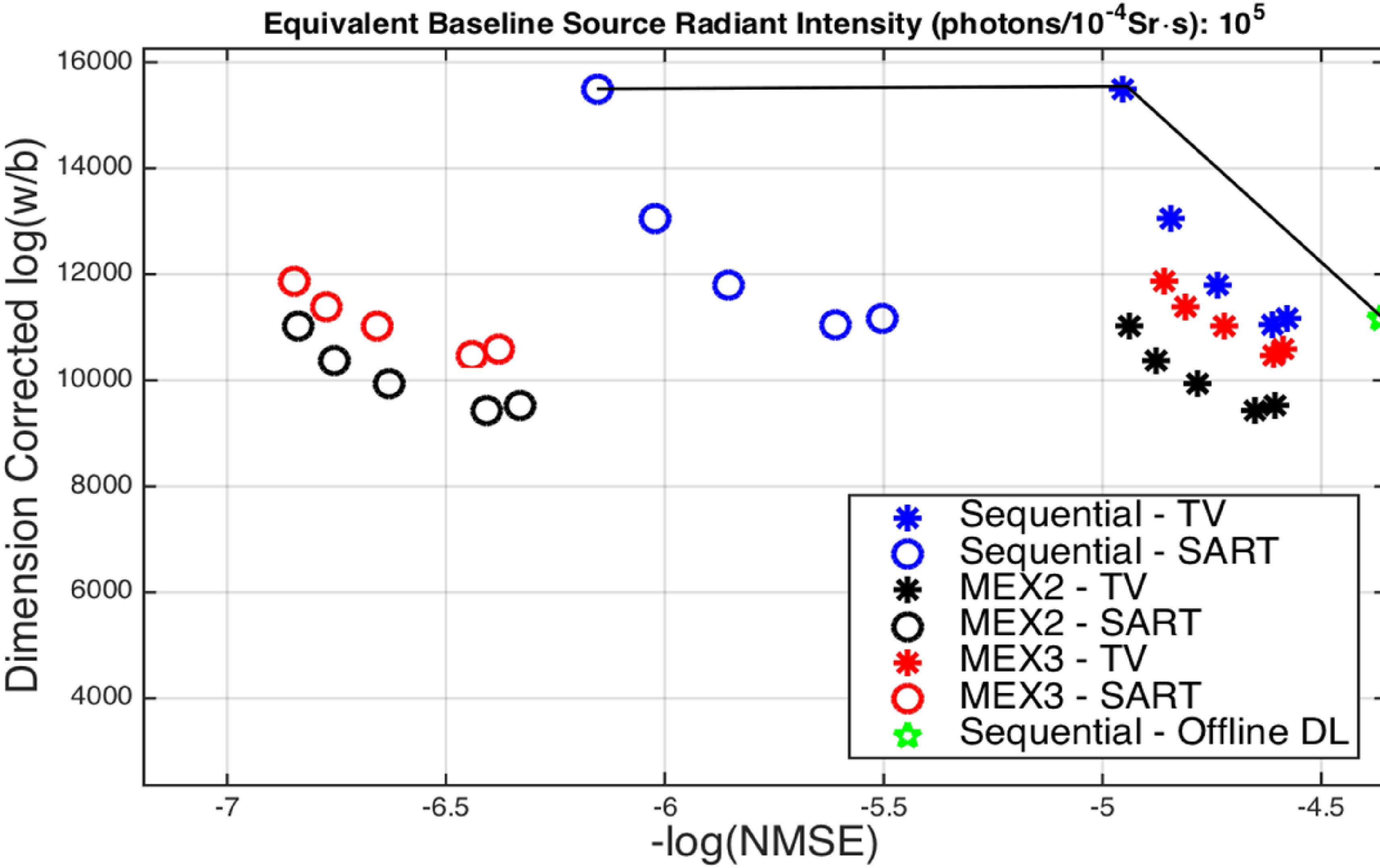
Multi-source, fixed gantry, rectangular CT system.

16,8,4,2,1 spectral bin scans used.

Sequential and Multiplexed Scanning

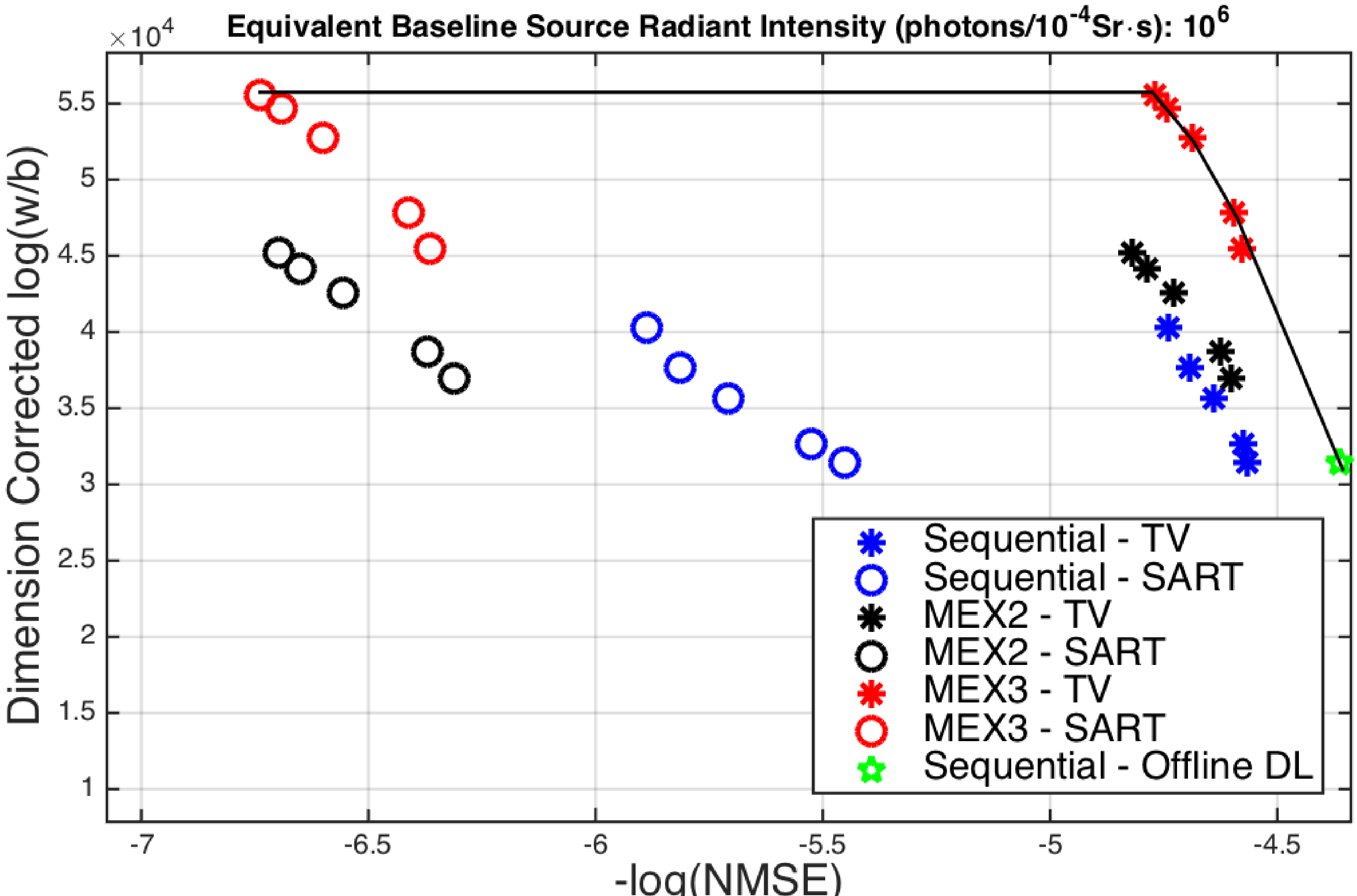


EDIT Curves



Low SNR Region

EDIT Curves



High SNR Region

Conclusion

- EDIT curves can be used to make design decisions.
- At low SNR Sequential scanning outperforms multiplexed scanning.
- Spectral binning reduces estimation performance but increases detection performance at these SNR levels.

Future Work

- Offline DL provides the best estimation but is computational intensive compared to simpler methods.
- Adjust EDIT curve to consider operator performance with estimation information.
- Use EDIT to determine the best tunable system parameters.
- Spectral binning reduces estimation performance but increases detection performance at these SNR levels.

Acknowledgments

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SPAWAR SSC Pacific

References

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How do we define the estimate for joint tasks?

We consider the case where the parameter vector θ to be estimated is associated with objects from both classes and that what distinguishes the two classes is the prior distribution on this parameter.

Basis Equations:

Cost Matrix: $\begin{bmatrix} C_{00} & C_{01} \\ C_{10} & C_{11} \end{bmatrix}$ Probabilities Matrix: $\begin{bmatrix} \Pr(TN) & \Pr(FN) \\ \Pr(FP) & \Pr(TP) \end{bmatrix}$

Average Cost: $C = P_0[C_{10}\Pr(FP) + C_{00}\Pr(TN)]$
$+P_1[C_{01}\Pr(FN) + C_{11}\Pr(TP)] + P_1\langle C(\hat{\theta}(g), \theta)\rangle$

Differential cost: $\tilde{C}_{10} = C_{10} - C_{00}; \tilde{C}_{01} = C_{01} - C_{11}$

$$B = P_0 C_{00} + P_1 C_{11}$$

New costs equation to Minimize:

$$\tilde{C} = P_0\tilde{C}_{10}\Pr(FP) + P_1\tilde{C}_{01}\Pr(FN) + P_0\langle C(\hat{\theta}(g), \theta)\rangle_{TN} + P_1\langle C(\hat{\theta}(g), \theta)\rangle_{TP}$$