



Tufts
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Compton Scatter Imaging

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So What, Who Cares

- What space/topic/area is being addressed?
 - X-ray-based baggage inspection
 - Nominally carryon but methods are more broadly applicable
- What problem have you solved?
 - Improve detection performance for severely limited view systems
- How have you solved the problem?
 - Similar to dual energy CT case:

Photoelectric + Compton → Material Maps → Detection

 - In limited view cases, DE image formation is at best challenging
 - We have development a new iterative reconstruction methods fusing traditional absorption data with Compton scatter photons

**Compton Scatter Photons = Additional Raypaths →
Improved Imaging → Improved Material Maps → Improved Detection**
- So what? Who cares?
 - Demonstrating the (potential) value of information typically thrown away
 - Ultimately increase P_d , decrease P_{fa} etc.



The Team



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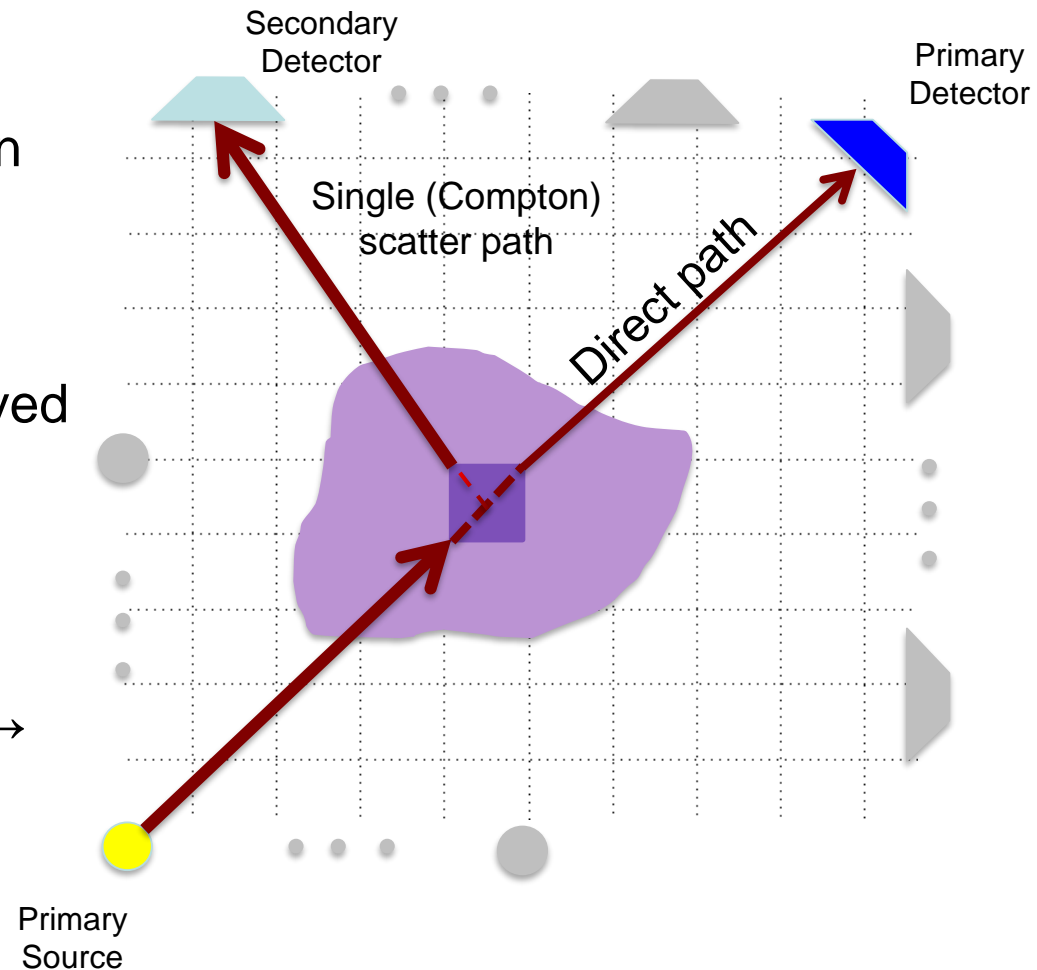


Brian Tracey



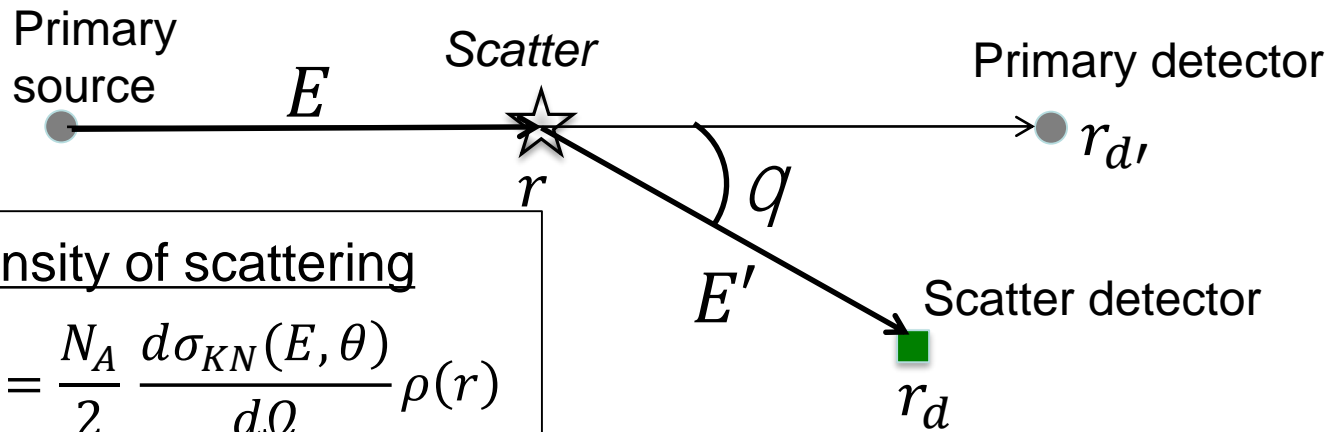
Background

- Ultimate goal: Improved detection
- Scenario of interest: few, fixed sources where traditional DE image formation will break down
- Approach:
 - Measure Compton Scatter = additional raypaths
 - Combined with energy resolved data (~100 few keV bins/detector)
- Rationale
 1. Improved ability to resolve photoelectric and density →
 2. Improved ability to characterize materials →
 3. Improve detection





Compton Scatter



Intensity of scattering

$$S(r, \theta, E) = \frac{N_A}{2} \frac{d\sigma_{KN}(E, \theta)}{d\Omega} \rho(r)$$

Change in direction

$$\cos(\theta(r, r_D, r_{D'})) = \frac{r - r_D}{|r - r_D|} \cdot \frac{r - r_{D'}}{|r - r_{D'}|}$$

Change in energy

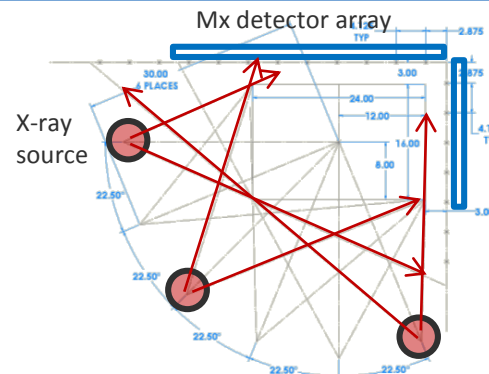
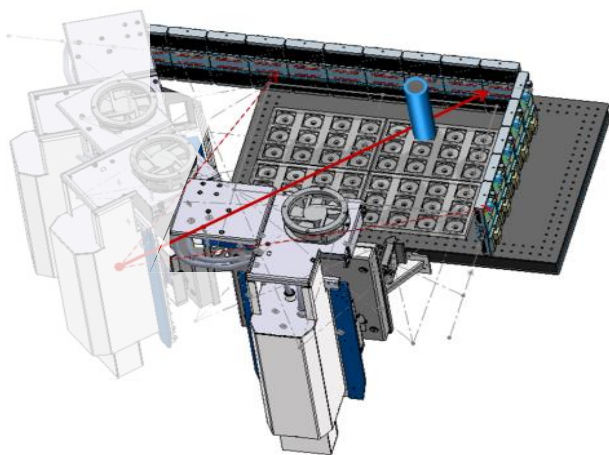
$$E' = \frac{E}{1 + \frac{E}{m_e c^2} (1 - \cos(\theta(r, r_D, r_{D'})))}$$

- From these physics we construct a computational model connecting maps of density and photoelectric absorption to energy resolved observation of attenuated and scattered photons.
- Use model as the basis for imaging



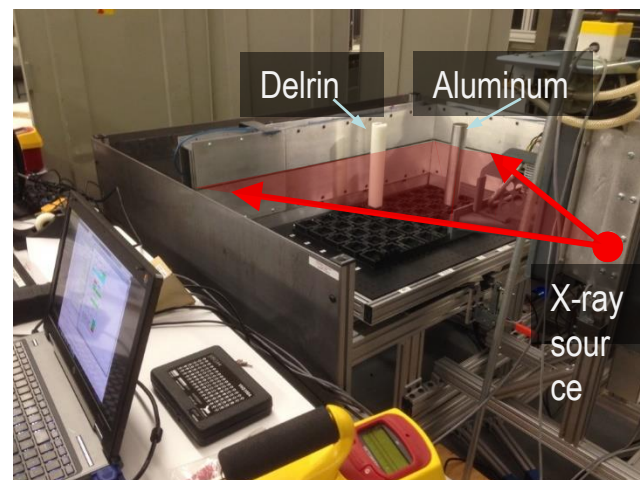
Is the model accurate?

Test Apparatus



Schematic top view of apparatus (end view of notional tunnel)

- Elementary target configuration consists of two image targets, each with a 2" diameter circular cross section:
 - Delrin (CH_2O) $Z_{\text{eff}} \sim 7$ $\rho = 1.4 \text{ g/cm}^3$
 - Aluminum (Al) $Z = 13$ $\rho = 2.7 \text{ g/cm}^3$

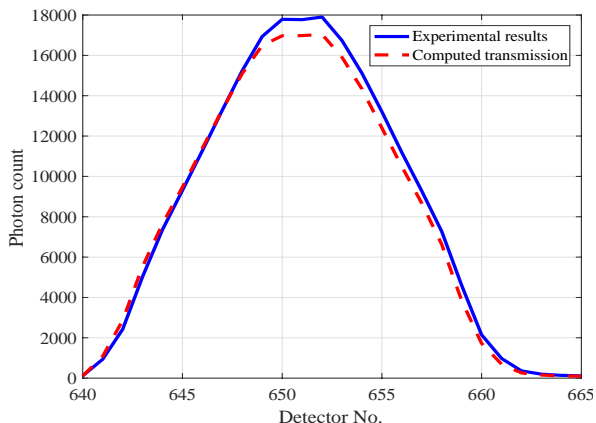




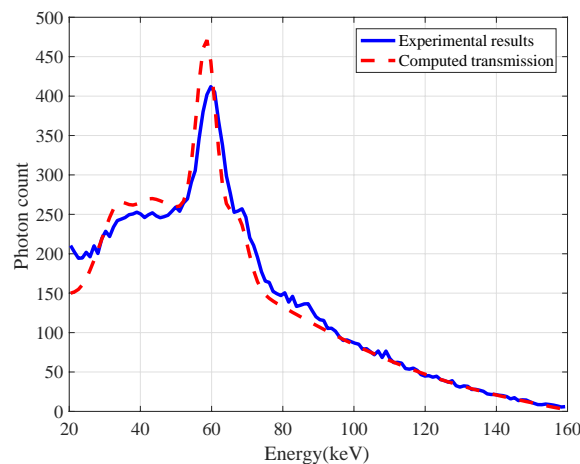
Tx model validation: Delrin and Aluminum, 2" cylinders

Delrin

Energy-summed, vs. beam



Multi-energy spectra, peak beam



Aluminum

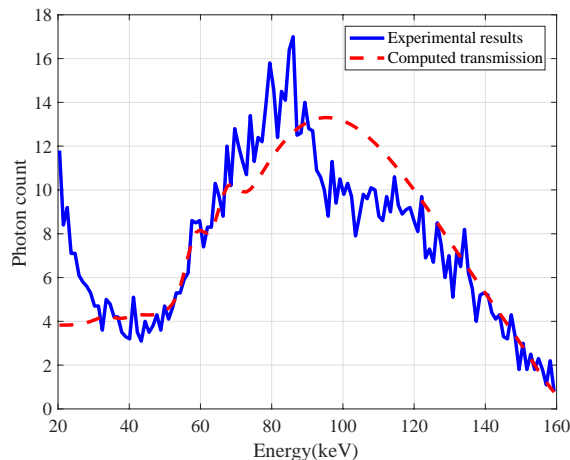
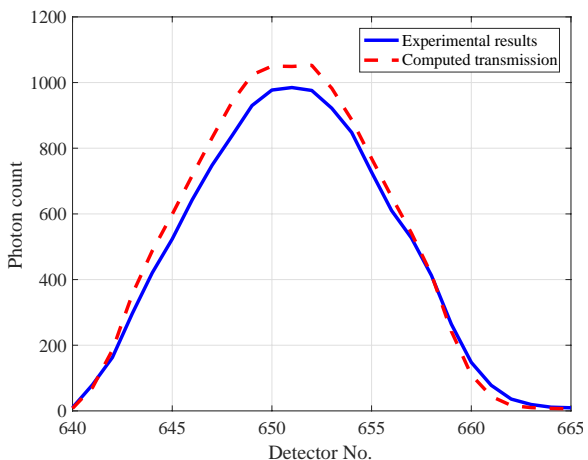


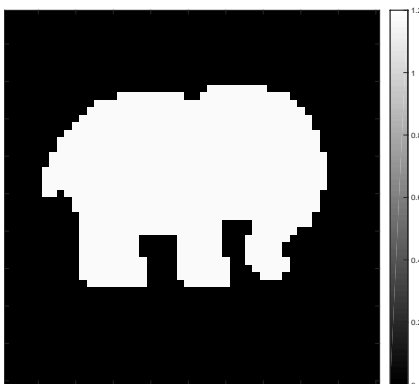
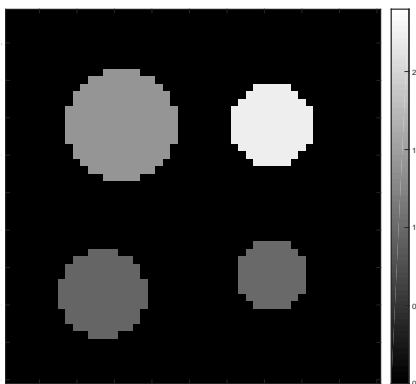


Image Formation: Initial Results

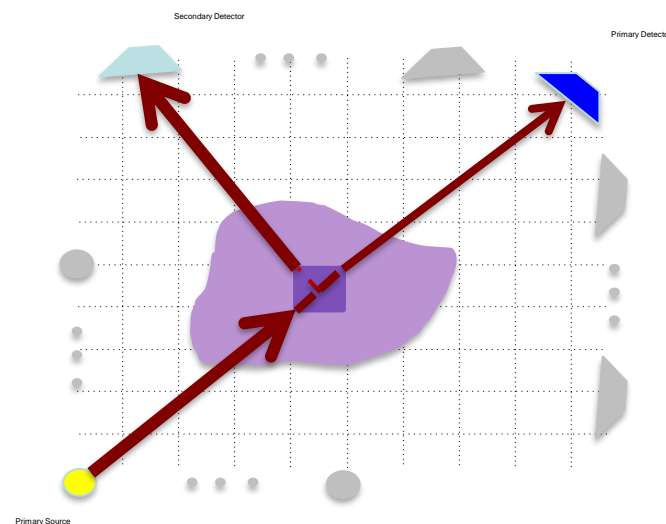
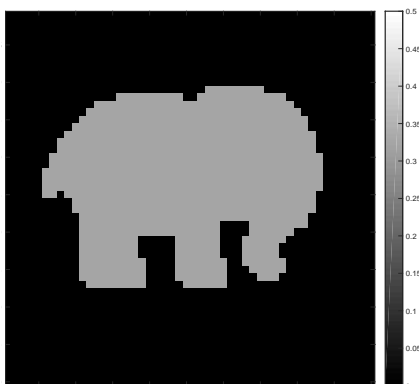
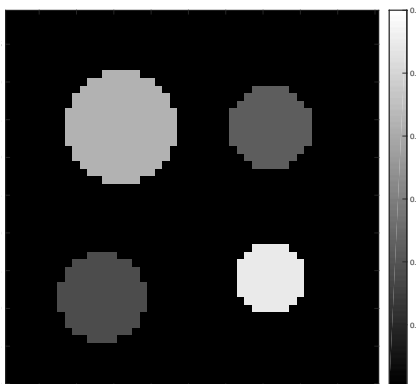
Phantom #1

Phantom #2

Density



Photoelectric

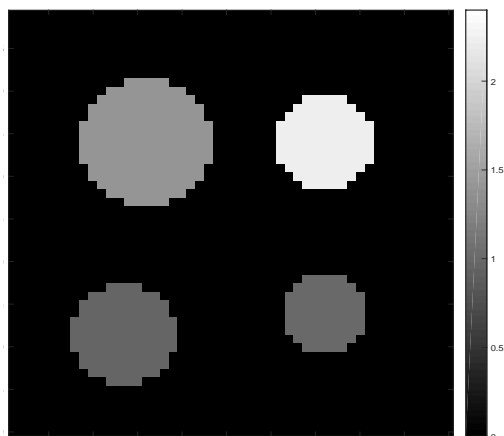




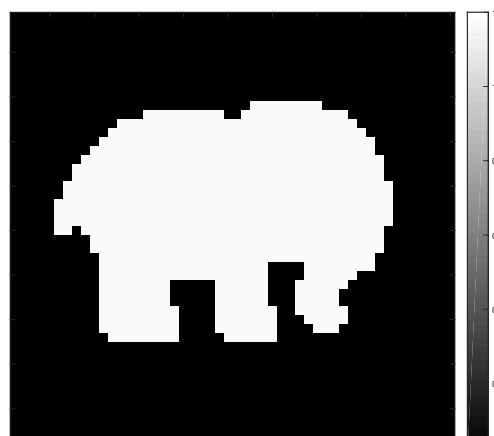
Initial Results

Density Reconstruction

Phantom #1

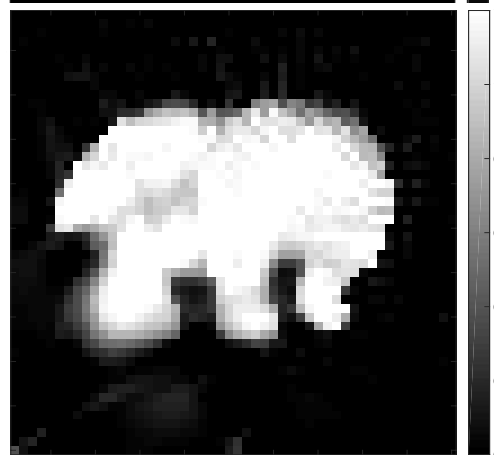
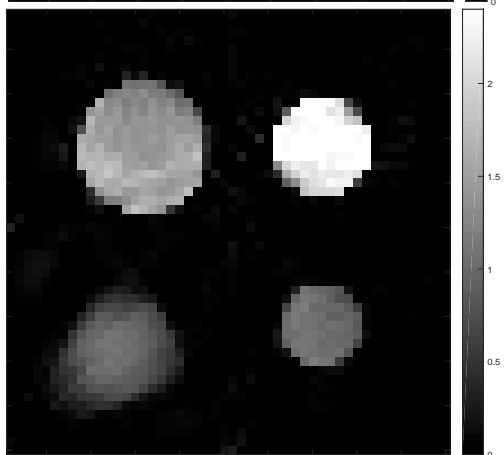


Phantom #2



Phantom #1:
0-2.4 g/cm

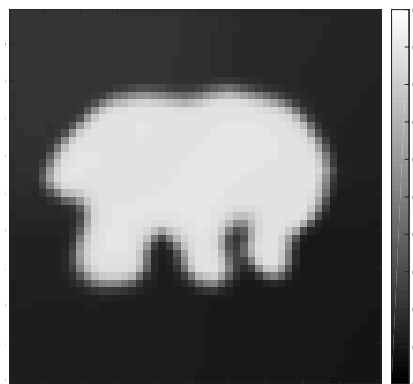
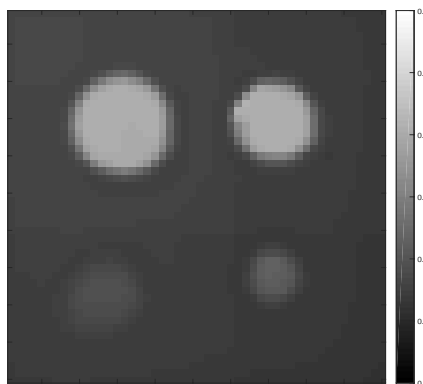
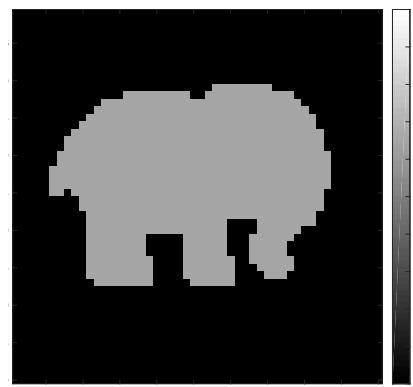
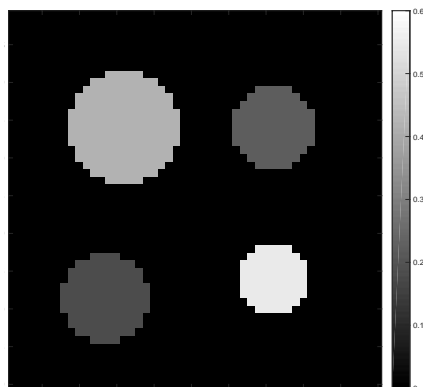
Phantom #2:
0-1.2 g/cm





Initial Results

Photoelectric Reconstruction Phantom #1 Phantom #2



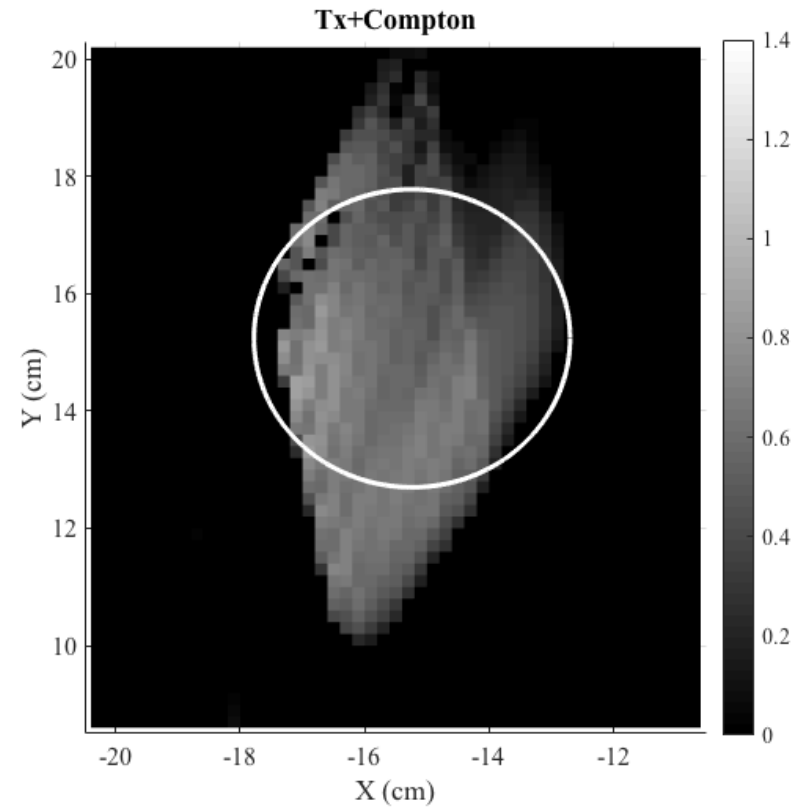
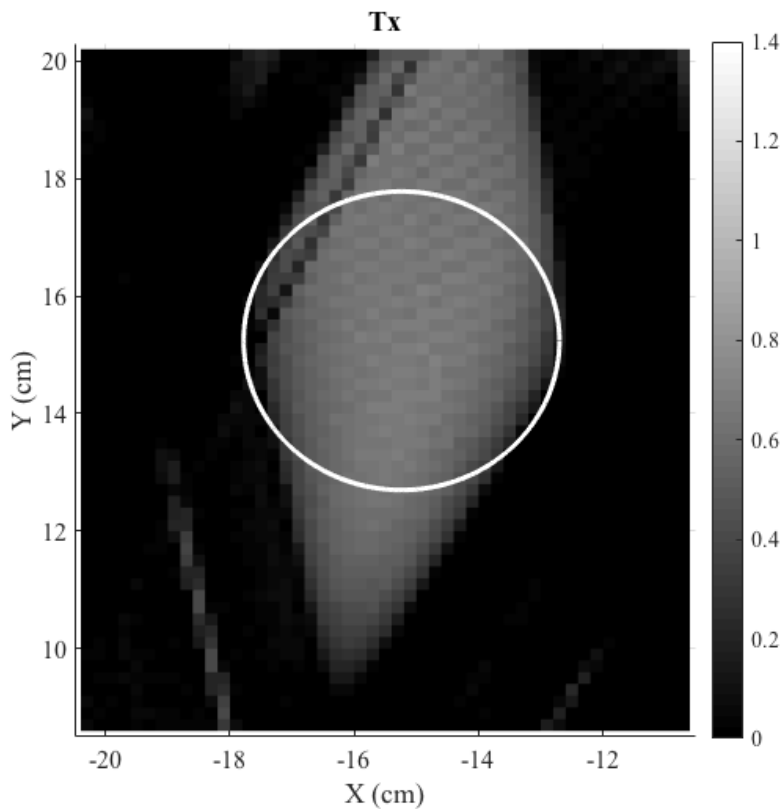
Phantoms 1: 0-.6 cm^{-1}

Phantom 3: 0-.5 cm^{-1}



Real Data: Initial Results

- Two views, 0 and 45 degrees source locations
- Low count data, averaging over 10 slots each with 0.1 sec observation

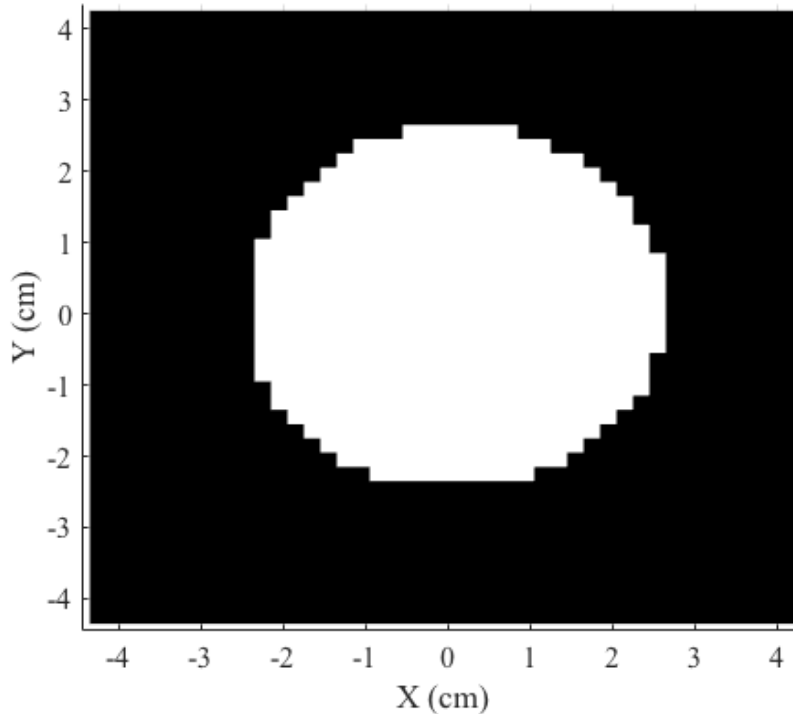




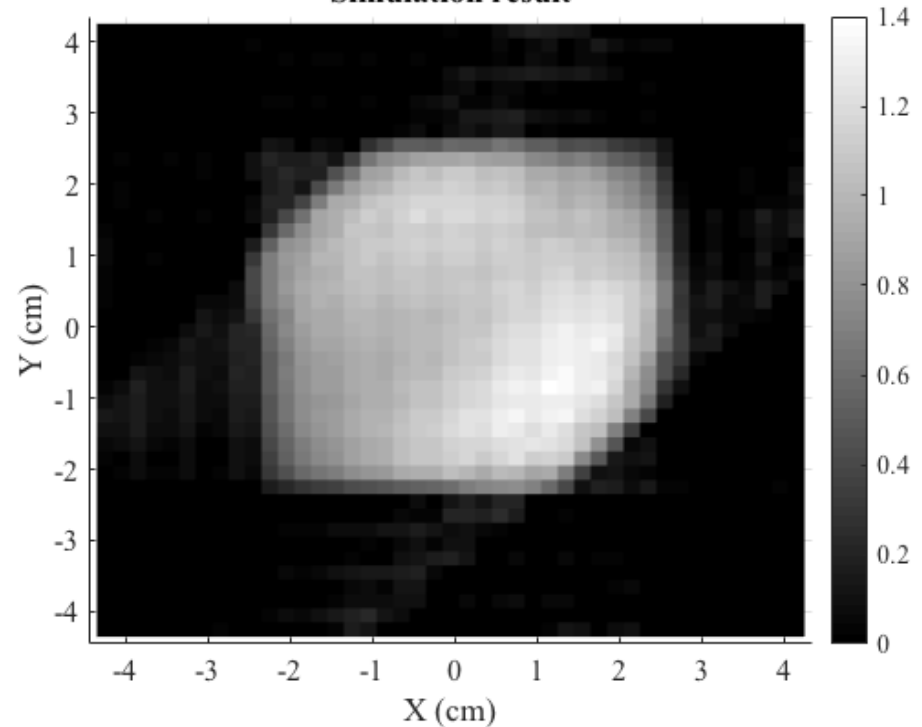
Looking Ahead to Better Data: Simulation Results

- Three views, 0, 45 and 90 degrees
- High count data are assumed

Actual profile



Simulation result





Conclusion

- Moving toward the conclusion that multi-energy scatter data can be fused with traditional absorption data to (substantially) improve imaging in limited view geometries
 - Certainly true in simulation.
 - Confident (at least ELM is) that this will be demonstrated from real data
- Materials ID to be explored in coming months
- Operationalization is not trivial
 - Scattered photons take time to collect.
 - Likely need to process scatter data in specific regions of interest
 - Computational burden is not small but methods are embarrassingly parallelizable
 - Work needed to understand trade-space comprised of computational architecture (CPA, FPGA, GPU), speed, and cost.
- May also be value in supporting effort in numerical linear algebra
- The story of this work is IMHO a nice example of how basic ALERT research can be moved out of the campus lab and toward actual application



BACKUP



Compton Scatter

- Hypothesis: Some energy leaving the main beam can be usefully recovered and ultimately improve detection performance
- Dominant process of interest here is Compton Scatter
- Inelastic scattering of an incoming X-ray photon by an electron



Discrete Compton Scatter Model

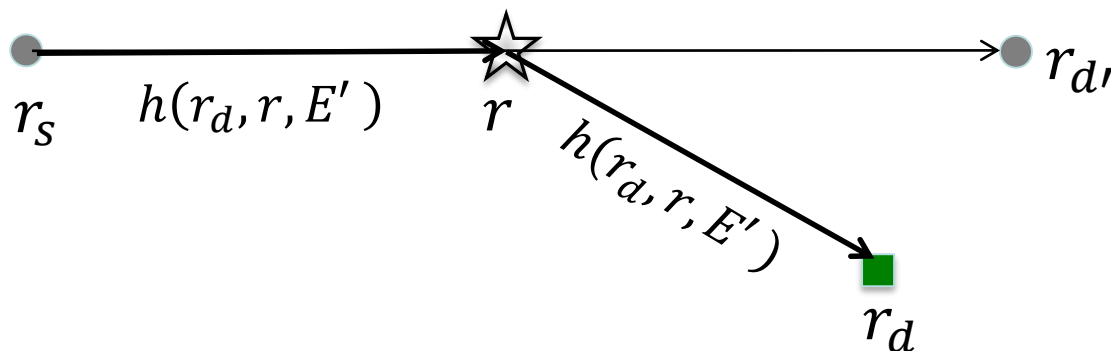
$$g = K(\rho, p)\rho + \mathcal{N}(0, \delta^2)$$

- Data vector aggregates information as a function of
 - Source-Primary Detector pair, $(r_s, r_{d'})$
 - Secondary detector: r_d
 - Energy: E'
- Nice structure:
 - Kind of linear in density
 - Will be exploited in processing
- For system with relatively few primary raypaths
 - Compton scatter gives many more “looks”
 - But signal strength lower. Either lower SNR or increased integration time
- Settle for additive white Gaussian noise for now. Poisson later.



Compton Scatter Model

$$\frac{N_A}{2} \frac{d\sigma_{KN}(E, \theta)}{d\Omega} \rho(r)$$



- Single scatter model
 - Propagate (attenuate) source to image point
 - Scatter at image point
 - Propagate image point to secondary detector

$$g(r_d, E') = \int I(E) \int h(r_d, r, E') \frac{N_A}{2} \frac{d\sigma_{KN}(E, \theta)}{d\Omega} h(r, r_s, E) \rho(r) dr dE$$

$$= \int K(r_d, r, E; \rho, p) \rho(r) dr$$



Compton Scatter Model

- Compton Scatter- Continuous form

$$g(\mathbf{r}_{D'}, E') = \int I_0(E_S) \left[\int h_2(\mathbf{r}_{D'}, \mathbf{r}, E') S(\mathbf{r}, \theta, E) h_1(\mathbf{r}, \mathbf{r}_S, E_S) l_{\mathbf{r}_D, \mathbf{r}_S}(\mathbf{r}) \rho(\mathbf{r}) d\mathbf{r} \right] dE_S$$



$$h_2(\mathbf{r}_{D'}, \mathbf{r}, E') = \Omega_D \exp(-\int \mu(\mathbf{r}', E') l_{\mathbf{r}_{D'}, \mathbf{r}}(\mathbf{r}') d\mathbf{r}')$$

$$h_1(\mathbf{r}_2, \mathbf{r}_1, E_S) = \exp(-\int \mu(\mathbf{r}', E_S) l_{\mathbf{r}_2, \mathbf{r}_1}(\mathbf{r}') d\mathbf{r}')$$

$$\mu(\mathbf{r}, E) = N_A \frac{z(\mathbf{r})}{A(\mathbf{r})} \rho(\mathbf{r}) f_{KN}(E) + p(\mathbf{r}) f_p(E)$$


- Compton Scatter- Discrete form

$$g = K(\rho, p)\rho + \mathcal{N}(0, \delta^2)$$


scattered data



discretized scattering system

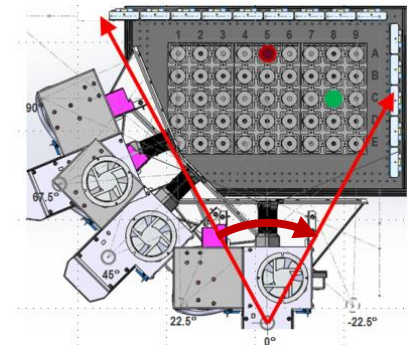


measurement noise



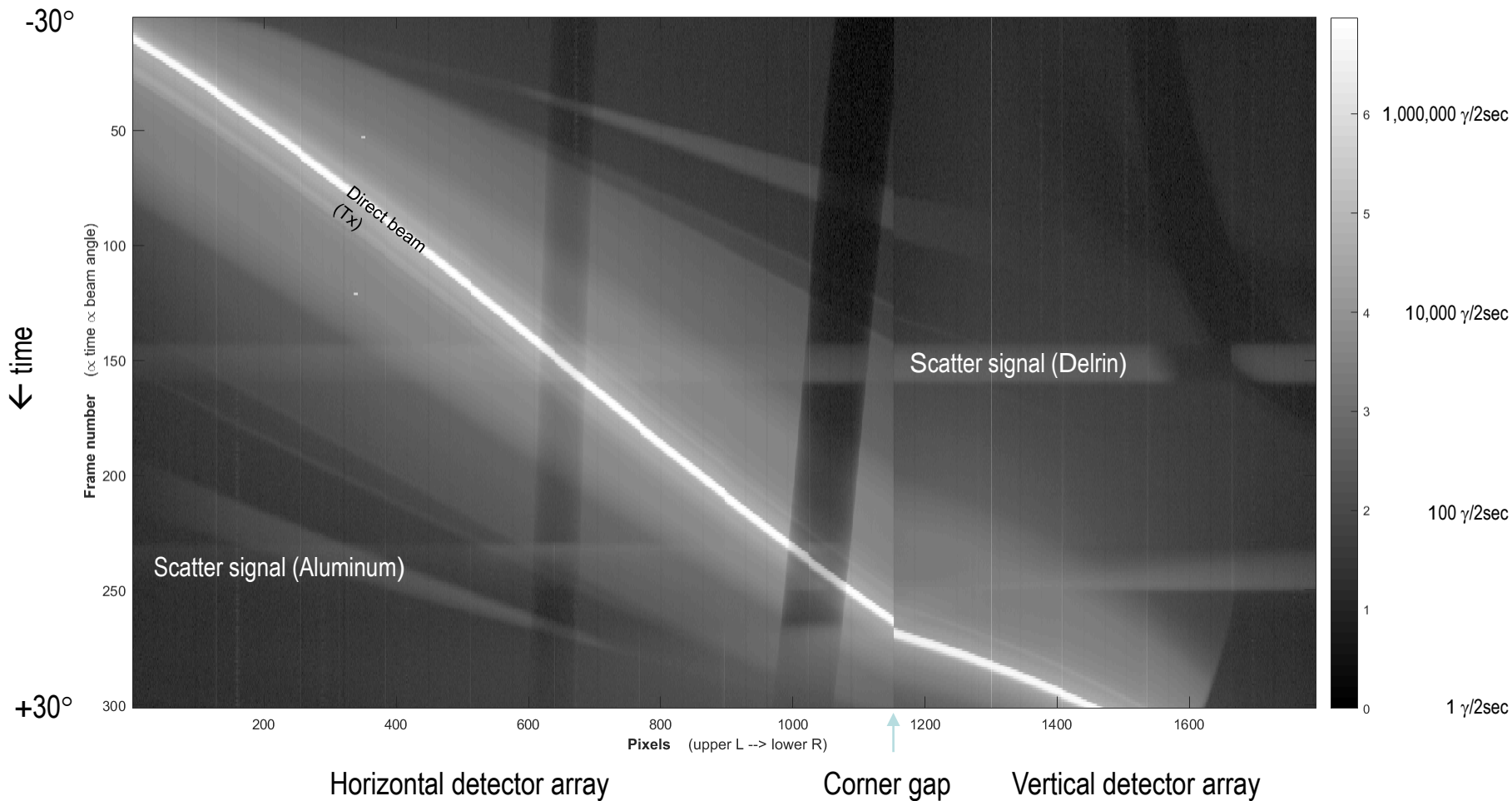


Amplified by Log10 scaling (grayscale)



Low signals emphasized

Color indicates counts/pixel per 2.0 sec time period (all energy channels summed)

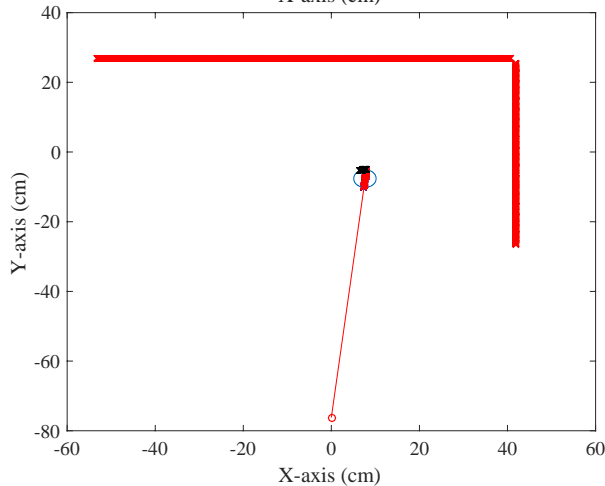
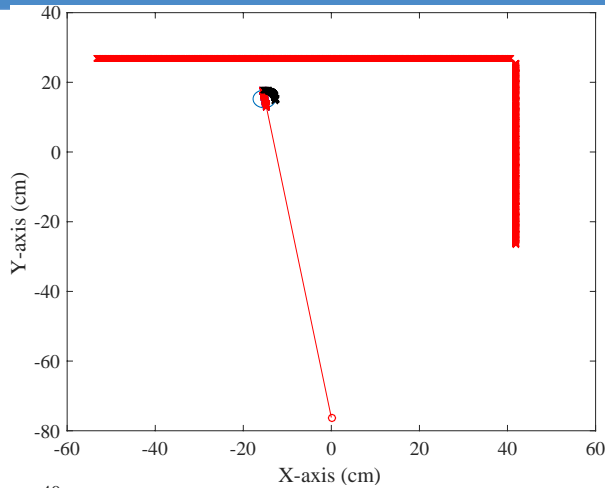




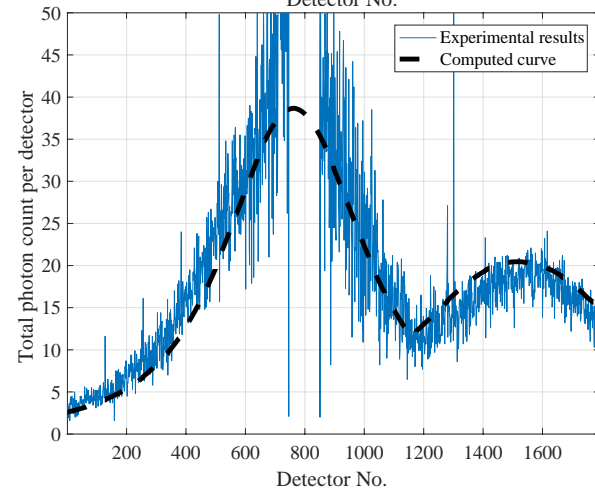
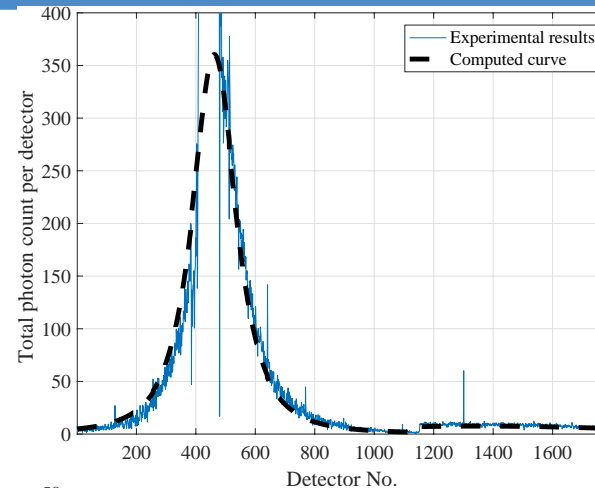
Scatter model validation: Varying target positions

Delrin target loc. D6 Delrin target loc. A3

Problem configuration



Energy-summed, vs. detector



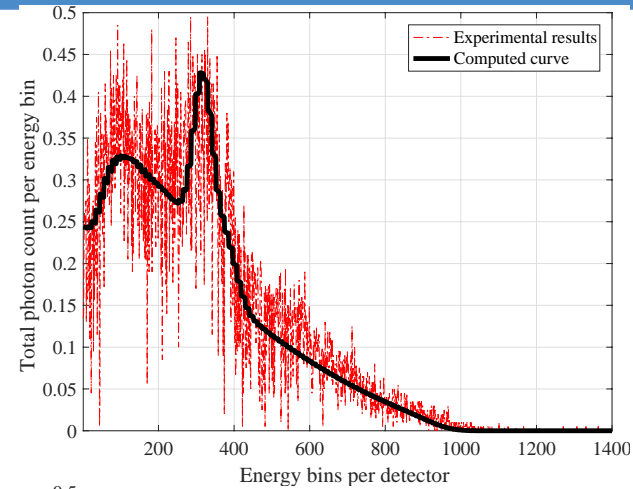
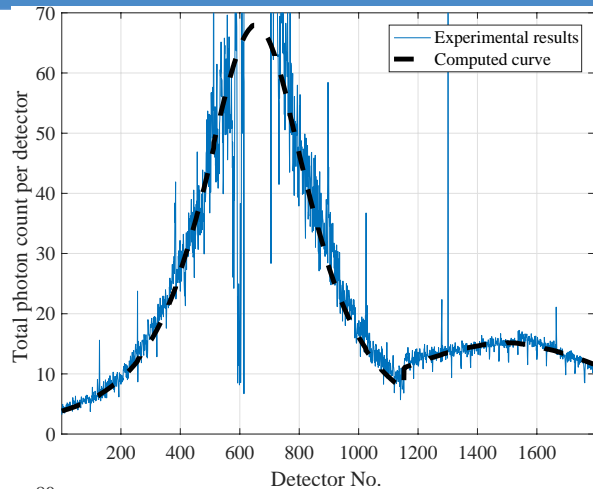
Underlying scattering is fairly isotropic; position cues are largely from solid angle effects



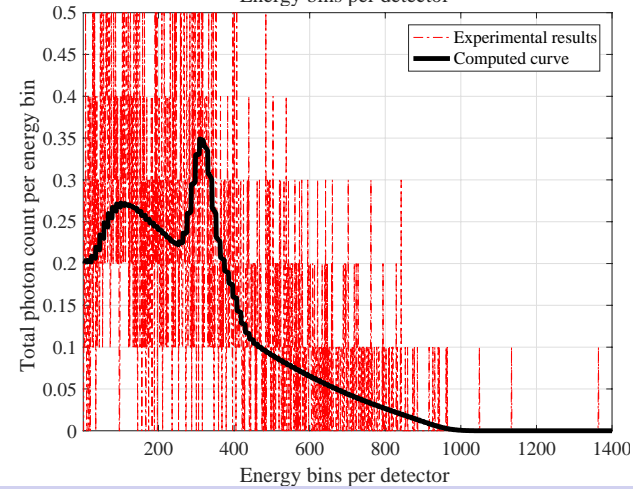
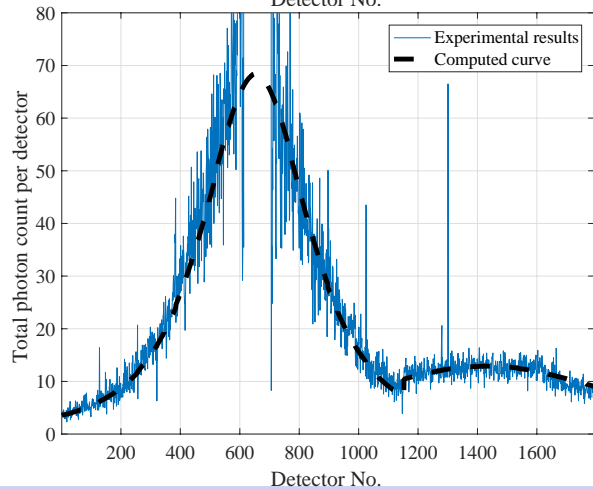
Scatter model validation: Spectra and effect of dwell time

Energy-summed, vs. detector Multi-energy spectra, peak beam

Graphite, 20
sec integration



HDPE, 1 sec
integration



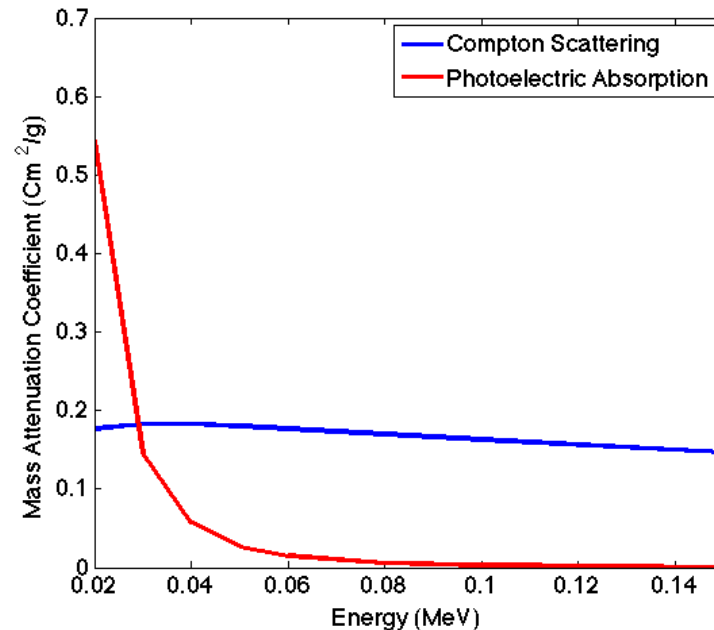
Model predicts spectral shape well; longer integration time reduces statistical noise



Inverse Problem

$$(\hat{\rho}, \hat{p}) = \underset{\rho, p}{\operatorname{argmin}} \underbrace{\|g - K(\rho, p)\rho\|_2^2}_{\text{data mismatch}} + \underbrace{R_\rho(\rho) + R_p(p)}_{\text{regularization}}$$

- Solution approach
 - Impact of photoelectric on data is small
 - Assume it can be ignored and first solve for density
 - After density recovered, estimate photoelectric
- Could iterate, but leave that for later





Density Reconstruction

$$\hat{\rho} = \underset{\rho}{\operatorname{argmin}} \|g - K(\rho, 0)\rho\|_2^2 + R_\rho(\rho)$$

Regularization

- Gradient-based

$$R_\rho(\rho) = \lambda_\rho \|L\rho\|_2^2$$

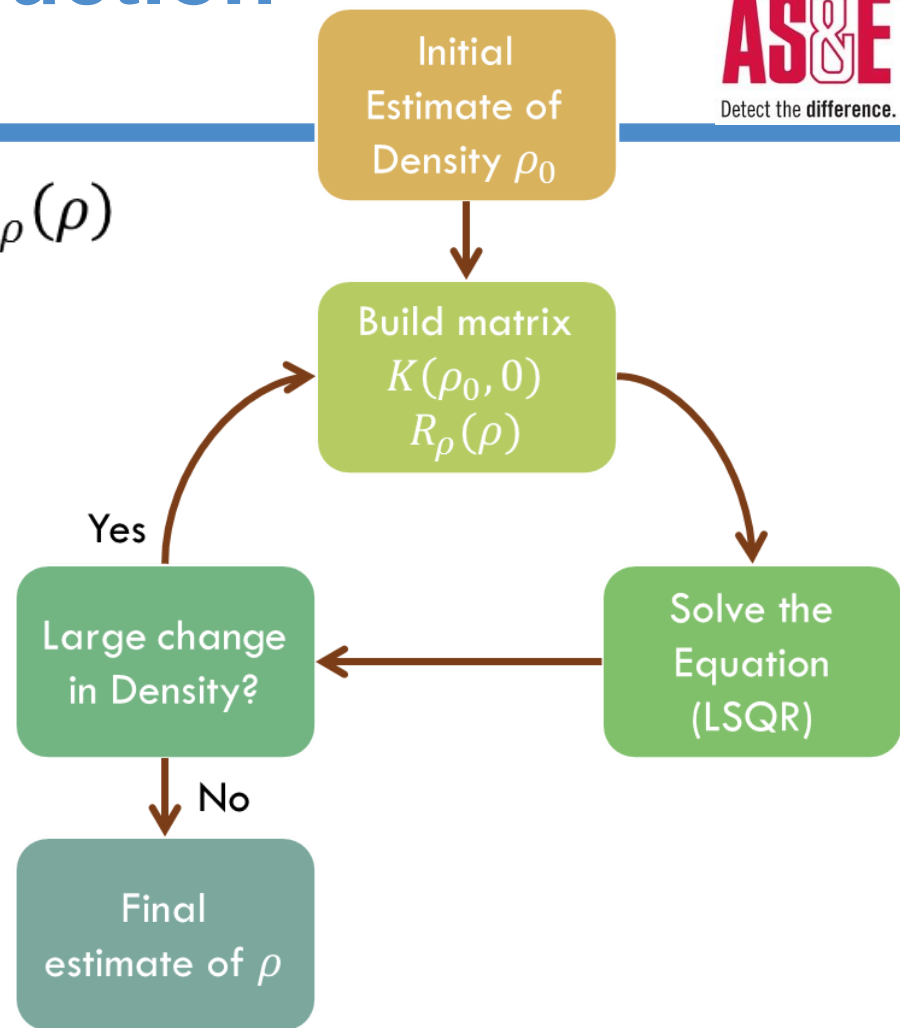
- Iterative Edge-Enhancing [1]

$$R_{\rho,\ell}(\rho) = \lambda_{\rho,\ell} \|D^{(\ell)}L\rho\|_2^2$$

- All λ_ρ chosen to minimize MSE (Clearly needs to be changed)

Initial Guess

- Attenuation based CT images
- Constant background image

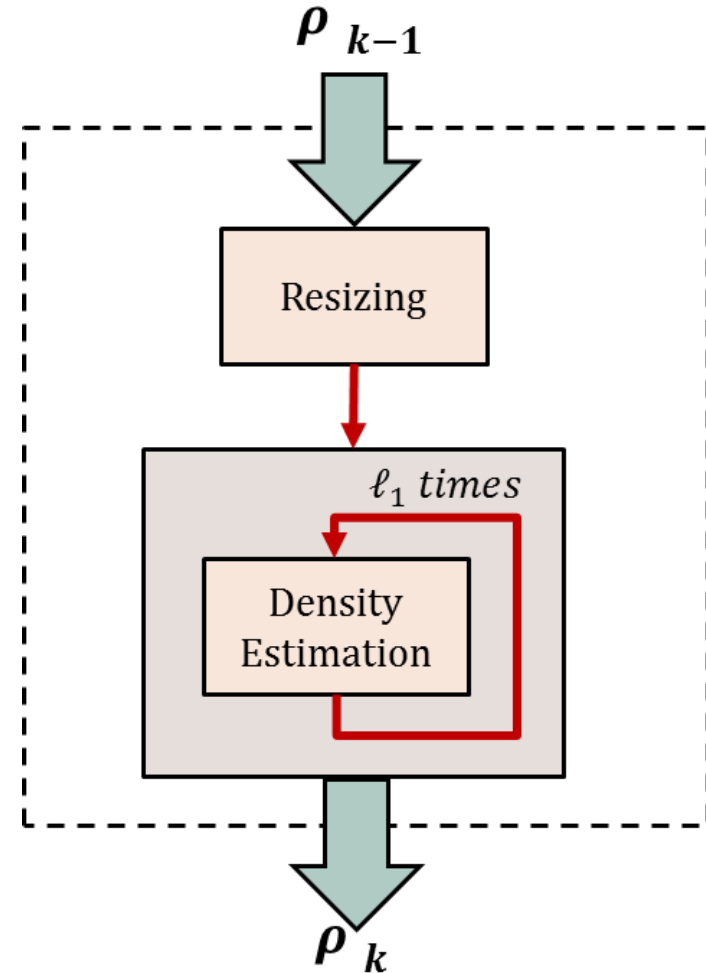


[1] Oguz Semerici, "Image Formation Methods for Dual Energy and Multi-Energy Computed Tomography," PhD Thesis, Dept. of ECE Tufts University, October 2012.



Multi-Scale Approach

- Initial efforts recovering density using fine scale grid of pixels did not work out so well.
- Multi-scale approach worked out much better
 1. Begin at coarse scale, $NR \times NC$, representation
 2. Initialized as a constant density image
 3. Estimate ρ
 4. Interpolate onto finer grid
 5. Goto 3 until fine enough
- Regularization parameter updated at every scale





Edge-Enhancing Regularization

- Gradient-based regularization penalizes all high differences even edges
- Edge-enhancing regularization de-emphasizes the smoothing for the edge locations in the image
- Diagonal elements on the weighting matrix determine whether a pixel belongs to the edge map
 - Closer to one : enforce smoothness
 - Closer to zero : should be preserved

$$R_{\rho,\ell}(\rho) = \lambda_{\rho,\ell} \left\| D^{(\ell)} L \rho \right\|_2^2$$

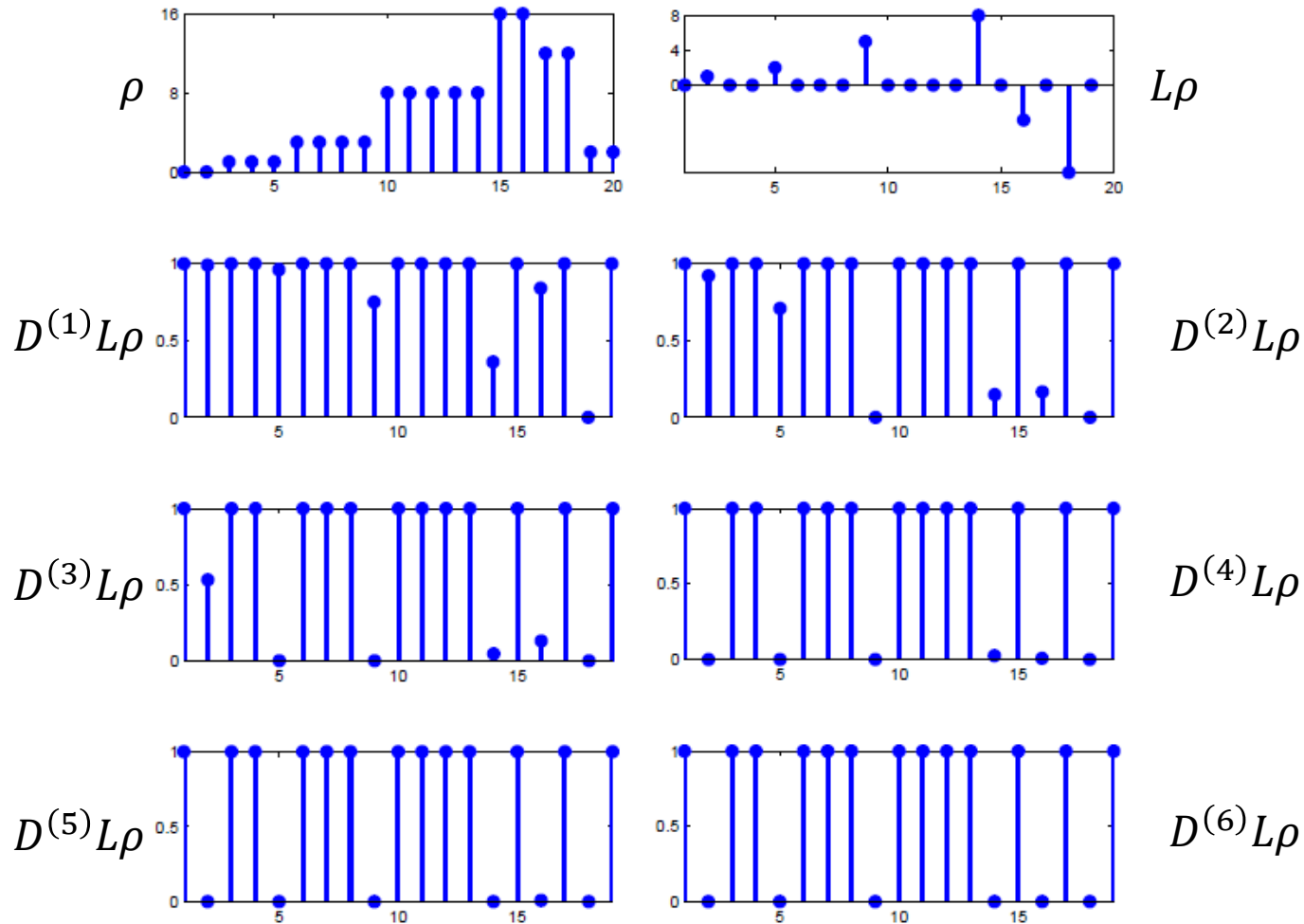
Inputs:

- $D^{(0)} = I$
 - **L gradient matrix**
 - **Estimate of ρ for $k = 0, 1, \dots$**
- 1: for iterations $k = 1, \dots$**
 - 2: Set $v = D^{(k-1)} L \rho_{k-1}$**
 - 3: Normalize v by setting $v \leftarrow v / \|v\|_\infty$**
 - 4: Map d to $[0, 1]$ by defining $d := 1 - v^p$**
 - 5: Define $D := \text{diag}(D)$**
 - 6: Update $D^{(k)} \leftarrow D D^{(k-1)}$**
 - 7: end**



Edge-Enhancing Regularization

$$R_{\rho,l} = \lambda_{\rho,l} \|D^{(l)} L\rho\|_2^2$$

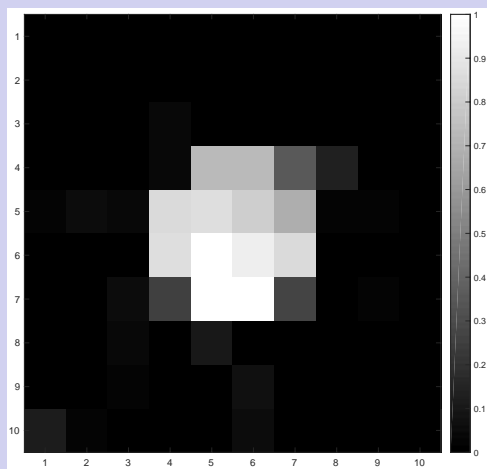




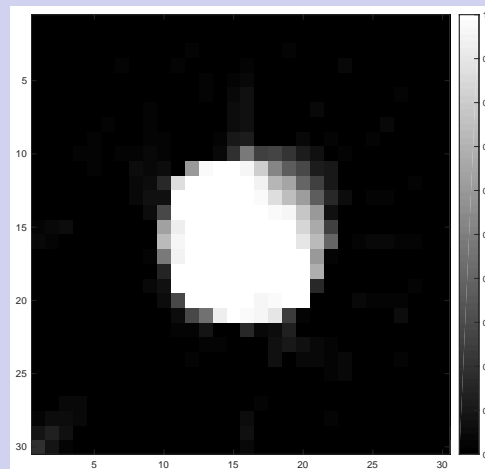
Simulation Results

Phantom #3

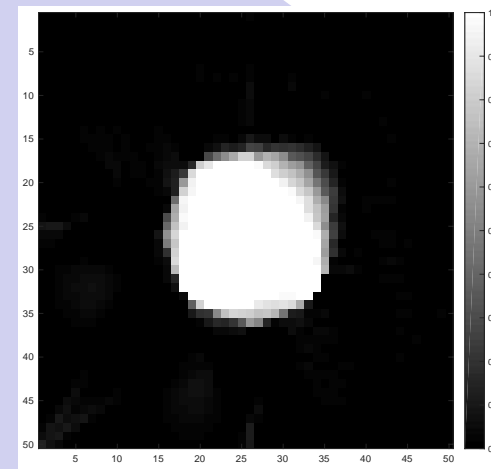
Density Estimation: Iterative Edge-Enhancing Regularization



Scale 1
10 × 10



Scale 3
30 × 30



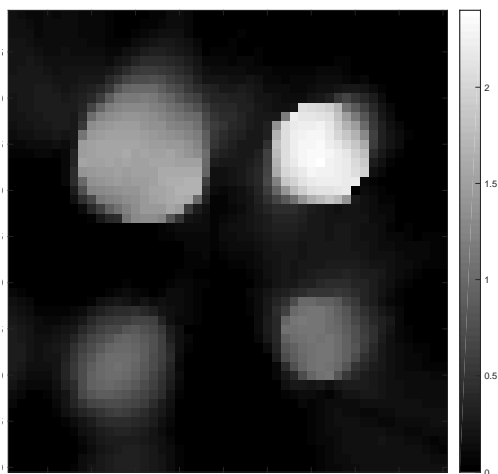
Scale 5
50 × 50



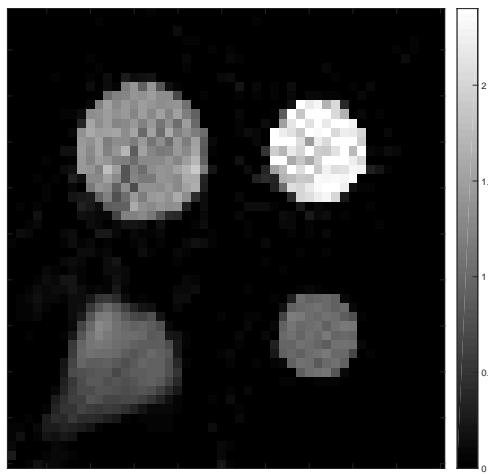
Initial Results

Density Reconstruction Value of Heterogeneous Data

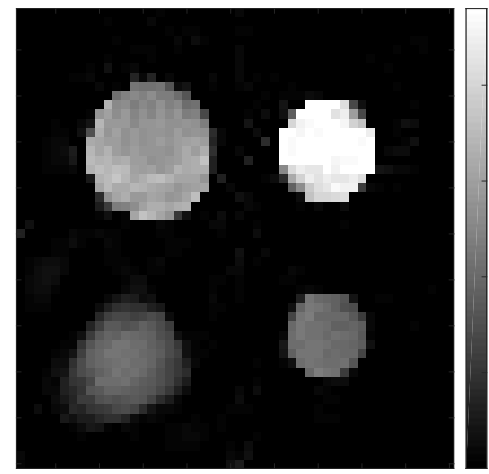
Only Attenuation Data



Only Scatter Data



Attenuation and
Scatter Data





Photoelectric Estimation

$$\hat{p} = \underset{p}{\operatorname{argmin}} \|g_{scat} - K_{scat}(\rho_t, p)\rho_t\|_2^2 + \|g_{att} - K_{att}(\rho_t, p)\|_2^2 + R_p(p)$$

- Joint attenuation and Compton Scatter inversion
- Non-linear least squares optimization problem
- Levenberg-Marquardt method [2]
- Patch-based non-local mean (NLM) regularization [3]
- Constant background image as initial guess

[2] D.W. Marquardt, "An algorithm for least-squares estimation of nonlinear parameters," Journal of the Society for Industrial and Applied Mathematics, pages 431–441, 1963.

[3] Brian H. Tracey and Eric L. Miller, "Stabilizing dual-energy X-ray computed tomography reconstructions using patch-based regularization," Inverse Problems, 31(10), 05004, September 2015



Patch-based Regularization

$$R_p(p) = R_{NLM}(p | \rho^{ref}) = \lambda_p \|(I - W)p\|_2^2$$

- Reduce noise artifacts
- Brings demising step into inversion process
- Calculates weighting matrix using density estimation as reference image

$$W(i, j) = \frac{1}{Z(i)} \exp\left(-\frac{\sum_{\delta \in \Delta} (\rho_{(i+\delta)}^{ref} - \rho_{(j+\delta)}^{ref})^2}{h^2}\right)$$

$$Z(i) = \sum_j W(i, j)$$

[2] D.W. Marquardt, "An algorithm for least-squares estimation of nonlinear parameters," Journal of the Society for Industrial and Applied Mathematics, pages 431–441, 1963.



Scaling Up: Parallel MPI Matlab code

- A parallel MPI Matlab code is developed to speed up the inversion process and reduce the memory cost
- The code distributes the algorithm such that each processing unit will process data from a single incident beam
- The code uses efficient memory storage where only the necessary beam-cell intersections are stored
- The memory is reduced by more than 20 times while the algorithm speed depends linearly on the number of processors