ALERT Task Order 3
Iterative Reconstruction using SIRT

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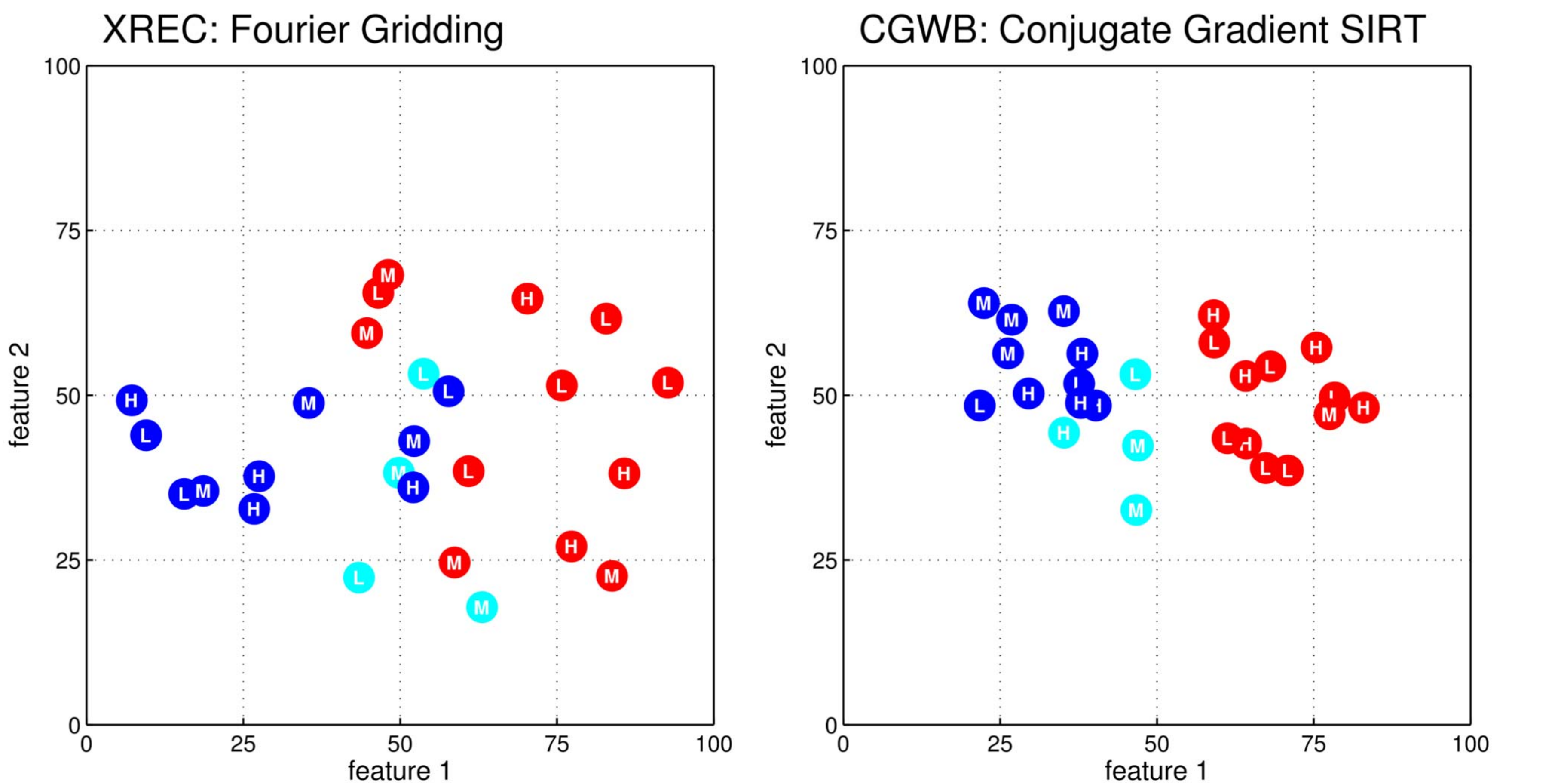
Clouds: Layout Overview

Side-by-side algorithm comparison for pair of features



Clutter: Low, Medium, High | **Water** | **Doped Water** | **Rubber Sheet**

Clouds: Feature Description

Metric analysis and clouds are based on image features

- Segmentation tools by Stratovan
- Ground trust mask features
  - μ: Mean pixel value [$0 \leq \ldots \leq \infty$]
  - σ: Standard deviation of intensities
  - Area 1 = 100 N/M [$0 \leq \ldots \leq 100$]

    N = #pixels in median ± 100 range

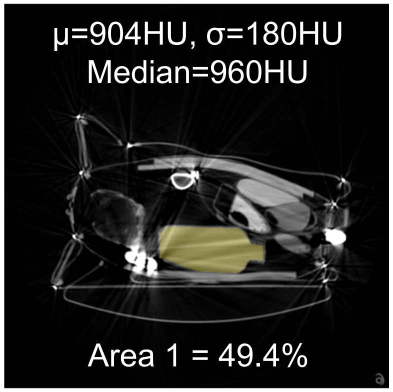
    M = #pixels in ground truth mask
- Tumbler segmentation features
  - Area 2 = (N-M)/M [$-1 \leq \ldots \leq +3$]

    N = #pixels in Tumbler mask

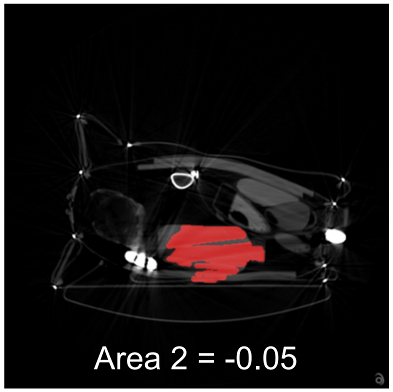
    M = #pixels in ground truth mask
- HU conversion (w/o calib. data)
  - HU = 1000 μ / μ_{water}

    μ_{water} = 0.2025 cm^{-1} (T. Gilat-Schmidt)
  - Air = 0 HU, water = 1000 HU

Ground truth mask

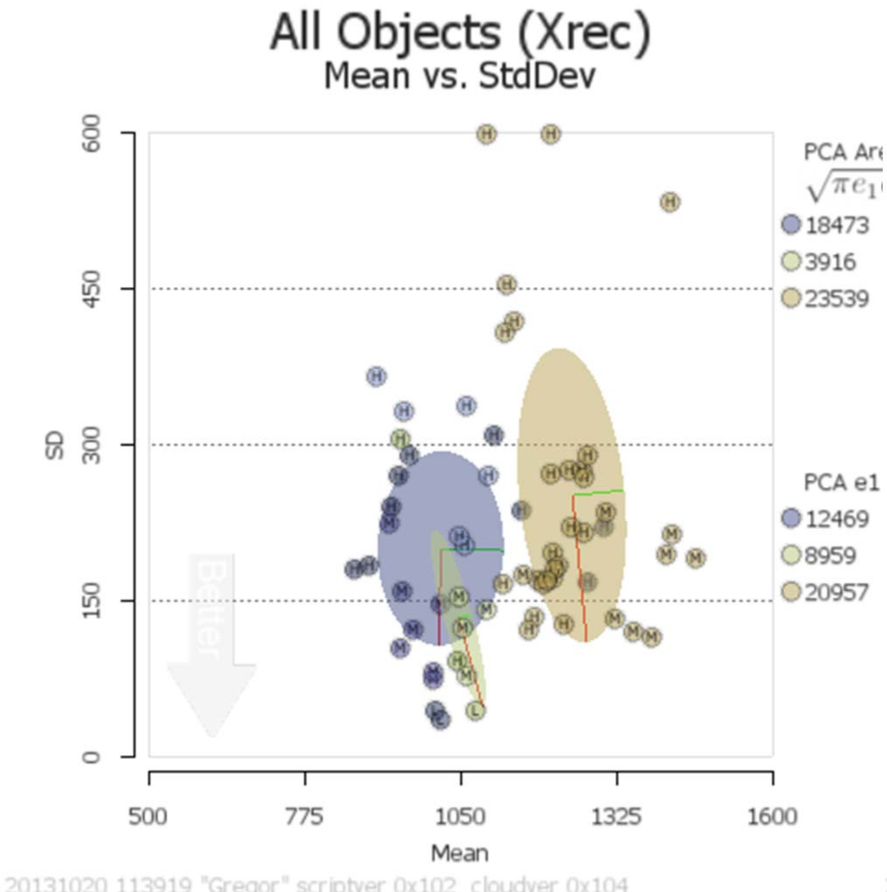


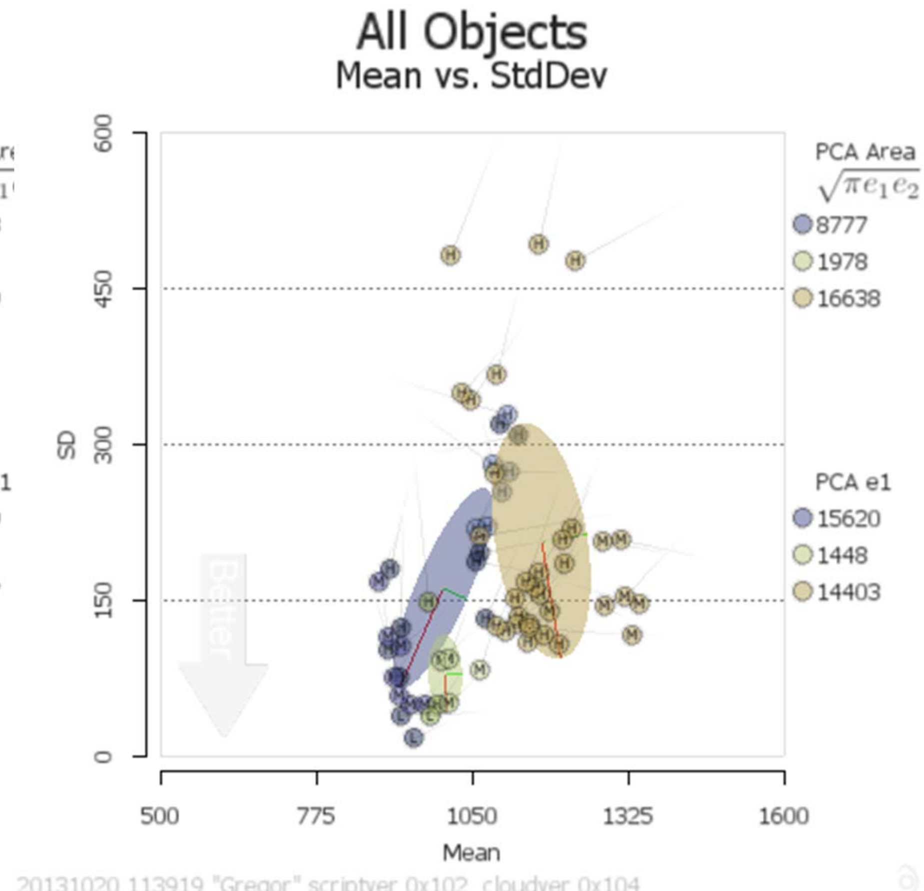
Tumbler segmentation



Clouds: Stratovan Version

Side-by-side algorithm comparison for pair of features





Clutter: Low, Medium, High

Principal Component Analysis: Shape

Mean Pixel Value Offset

Interpretation guide

- Without access to calibration data, different parameter setting yield different HU values
- To facilitate visual comparison, mean pixel values displayed relative to ensemble mean
- Negative value: “lower than overall mean” Positive value: “higher than overall mean”
- Differences are preserved, making material separation interpretation possible (ΔHU)

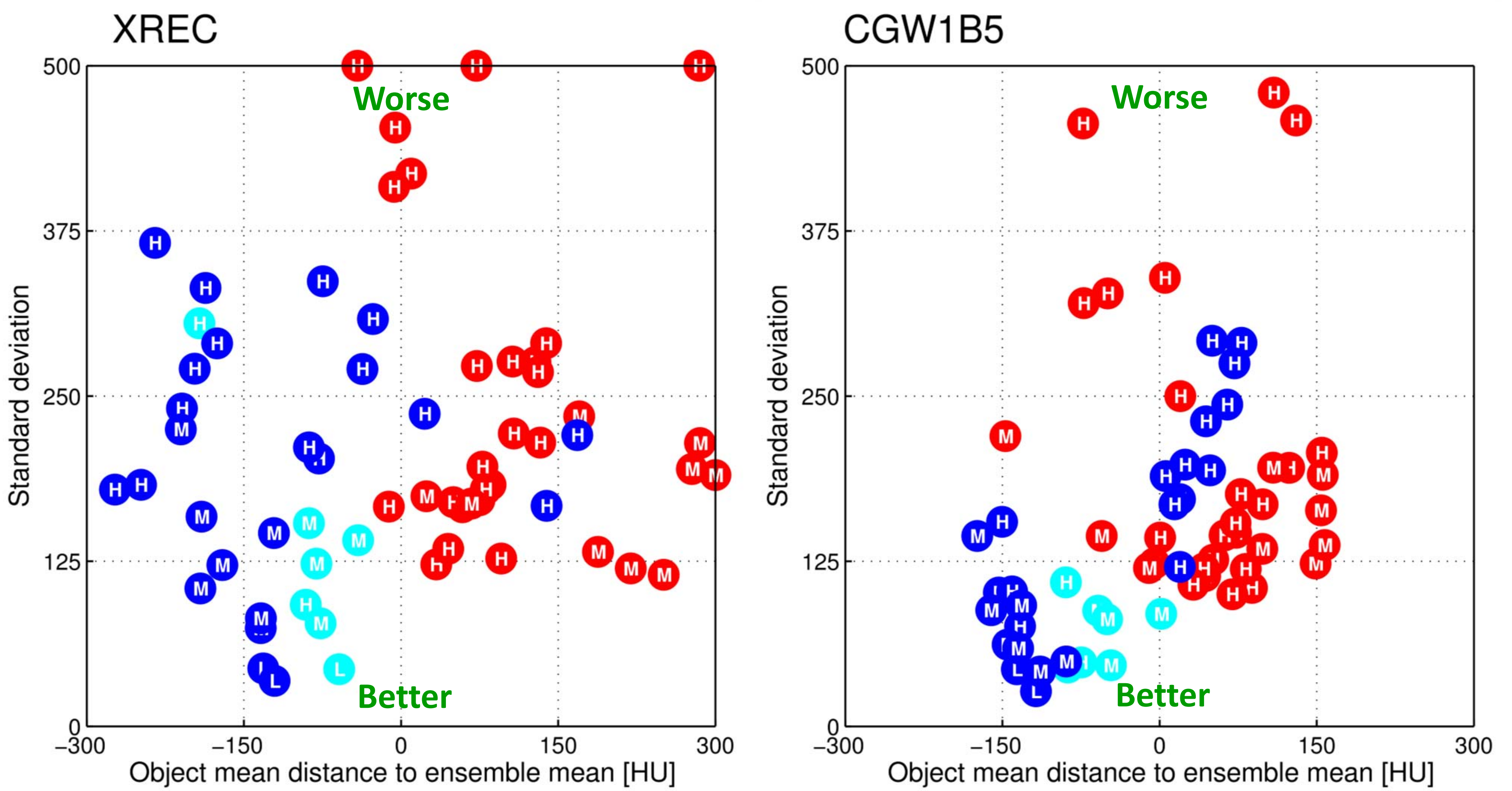
Tumbler Segmentation

Interpretation guide

- Warning: These results can be misleading
- Segmentation based on seed point chosen by Stratovan meaning object fragments neither produced nor fused as in a real ATR system
- Stacked rubber sheets inseparable – area recovery metric not only overestimated but also attributed to multiple rubber sheets
- View images when interpreting segmentations

Clouds: Mean vs Standard Dev.

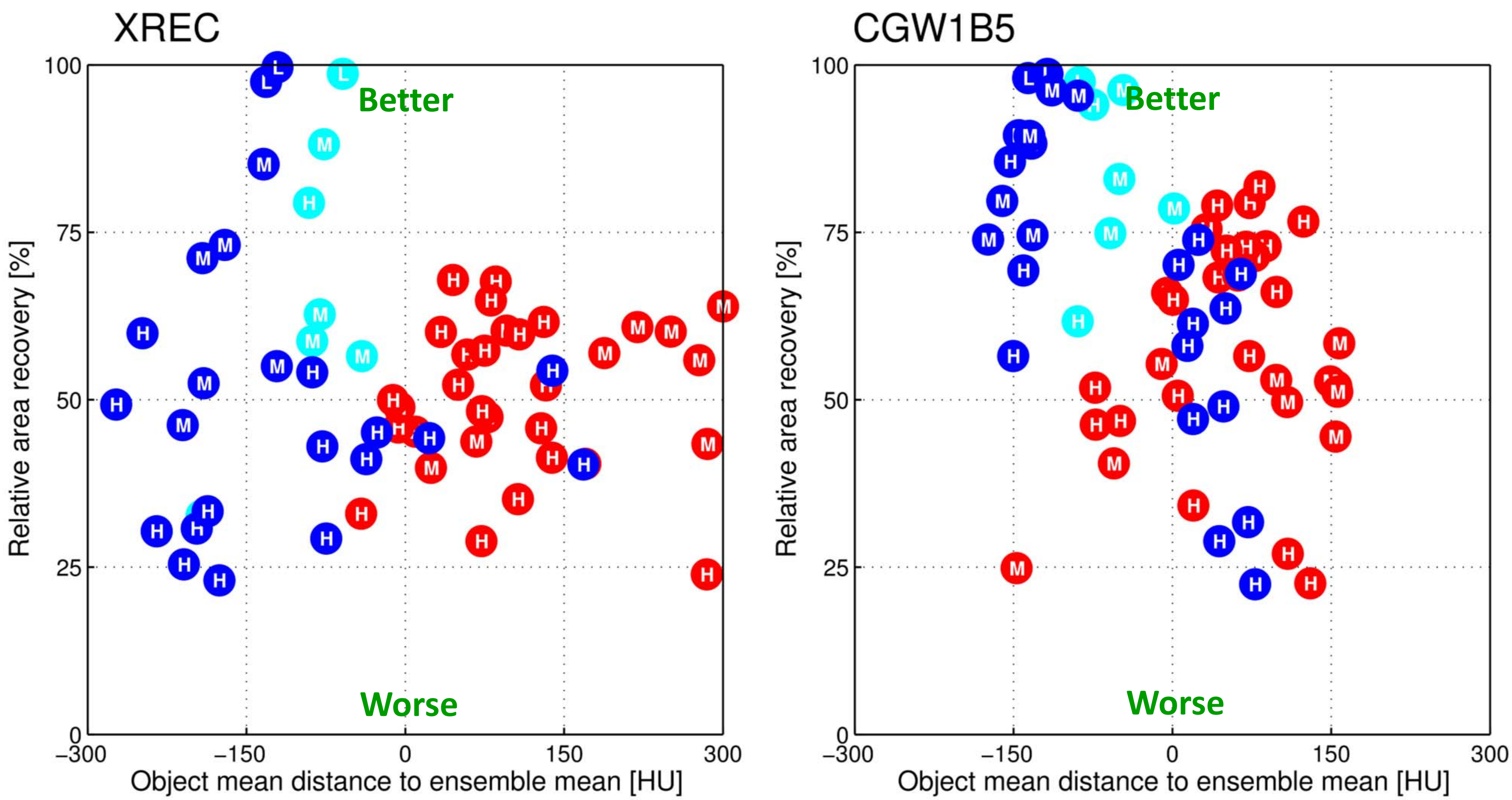
Mean and standard deviation of pixels in ground truth mask region



CGWB clouds are tighter than XREC. Standard deviation values are better.

Clouds: Mean vs Area 1

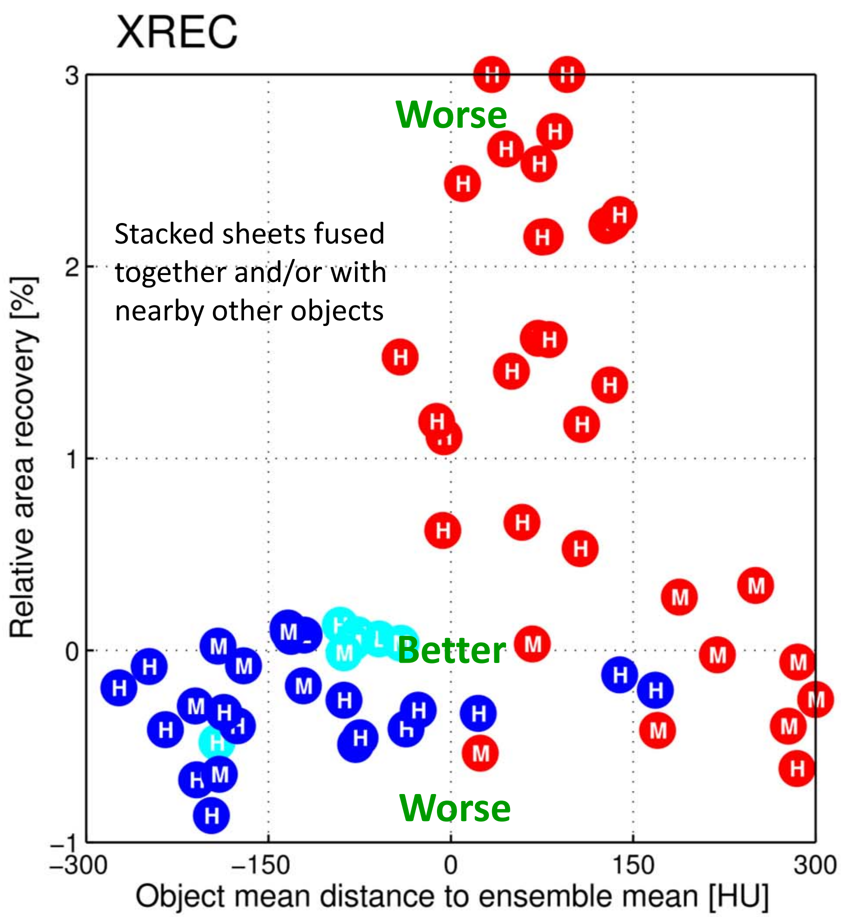
Num. pixels in median ± 100 HU range relative to ground truth mask size

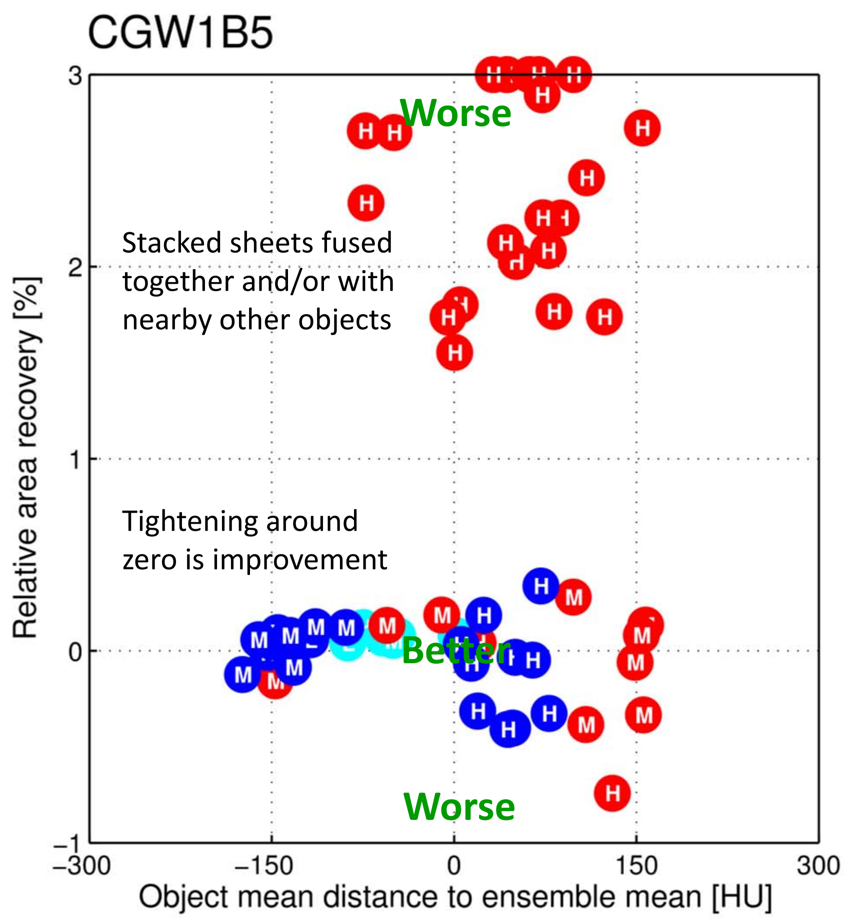


CGWB clouds are tighter than XREC. Relative area recovery is better.

Clouds: Mean vs Area 2

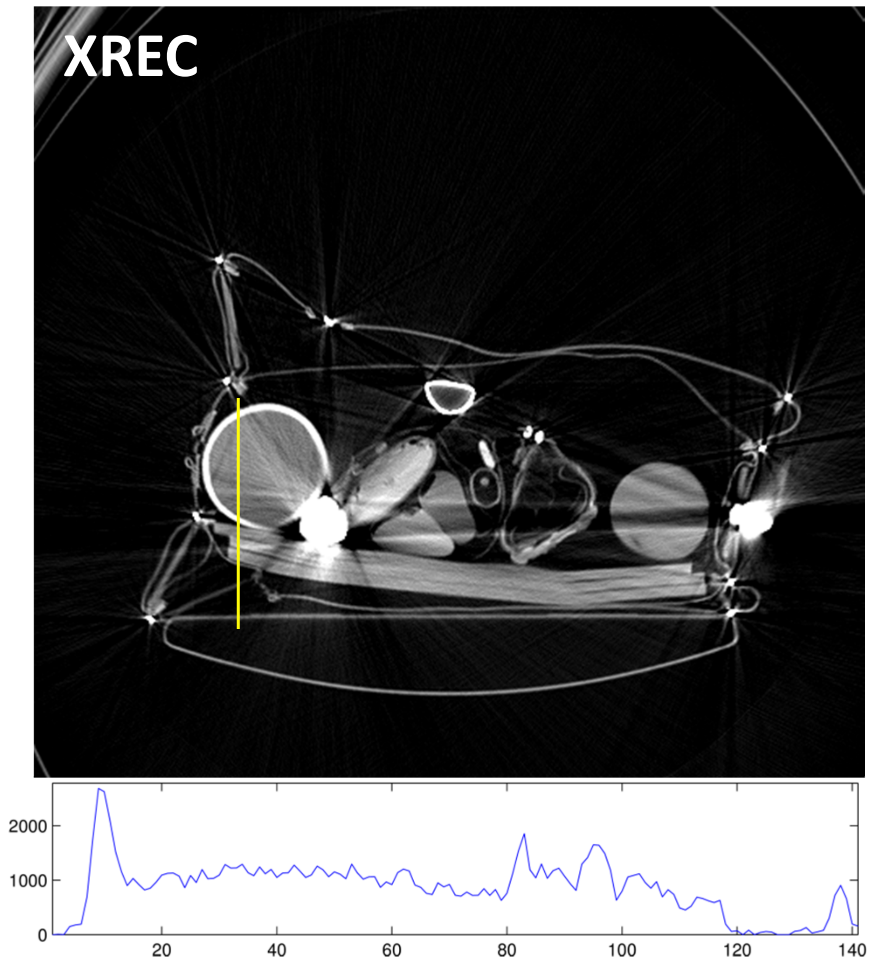
Num. pixels in Tumbler mask region relative to ground truth mask size

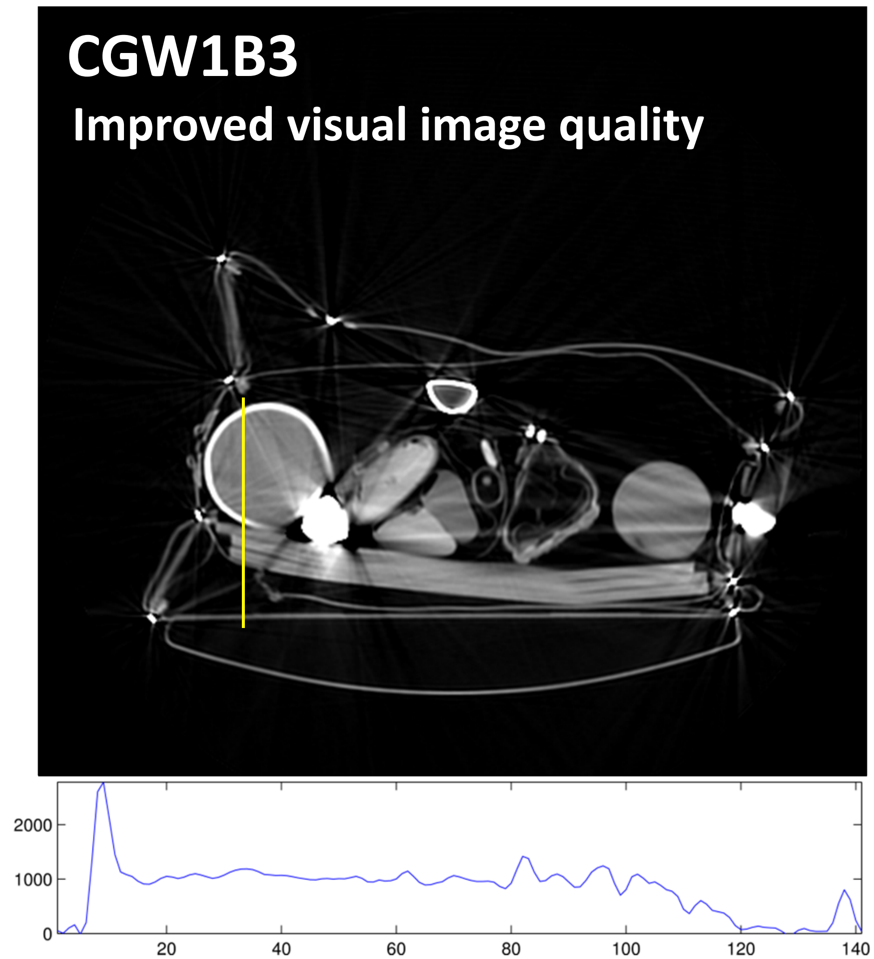




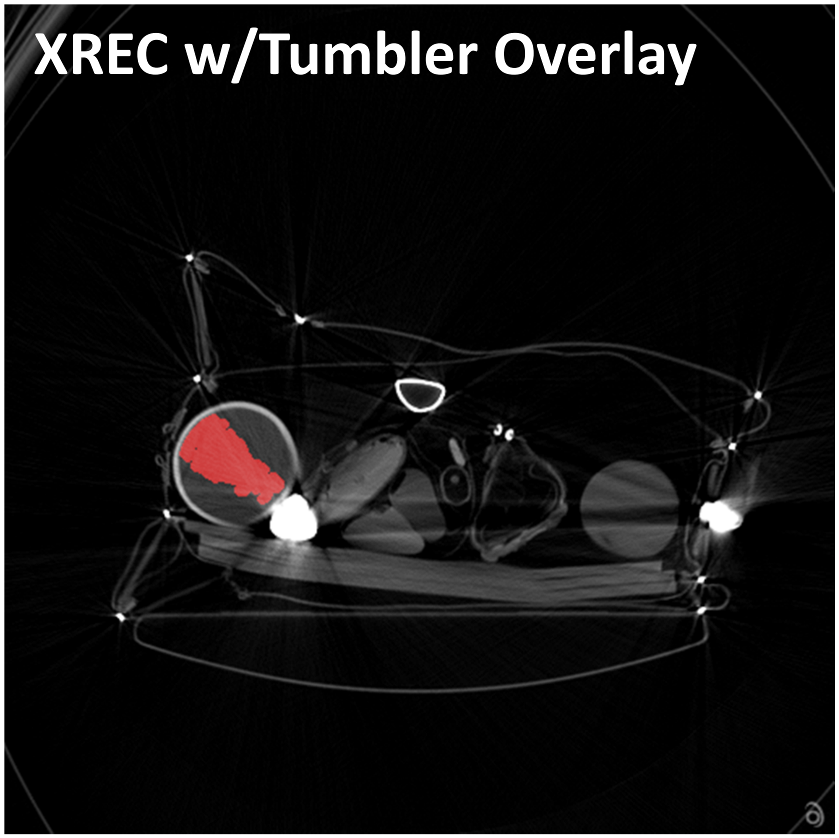
CGWB water area recovery better than XREC. Rubber sheets problematic.

Example: High Clutter 1 Slice 239

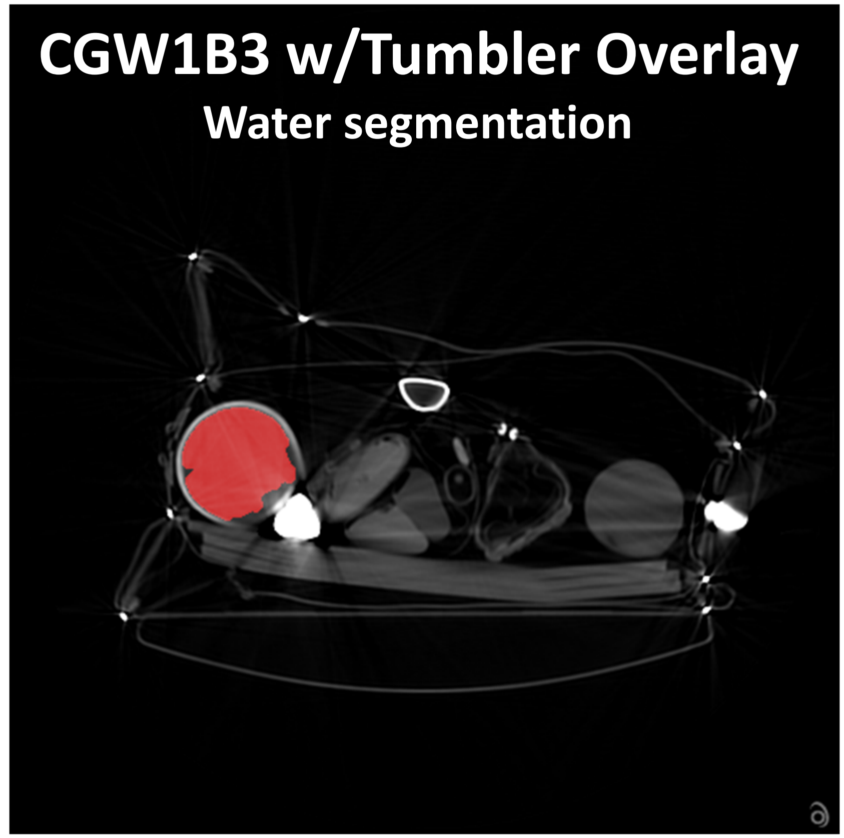




Example: High Clutter 1 Slice 239



XREC under-segments water objects.

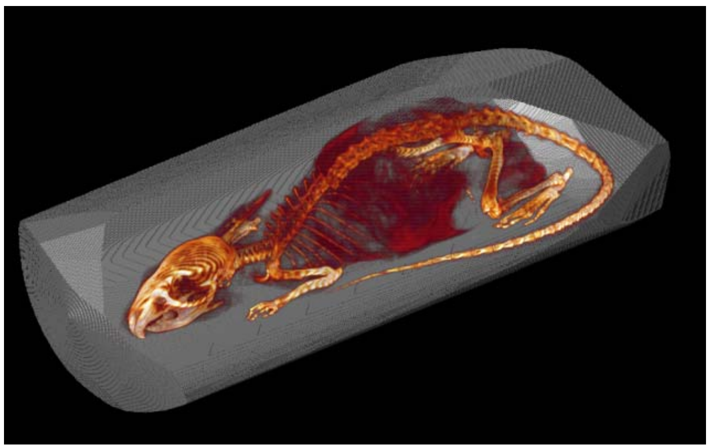


CGWB achieves better water segmentation.

Dr Gregor’s Imaging Experience

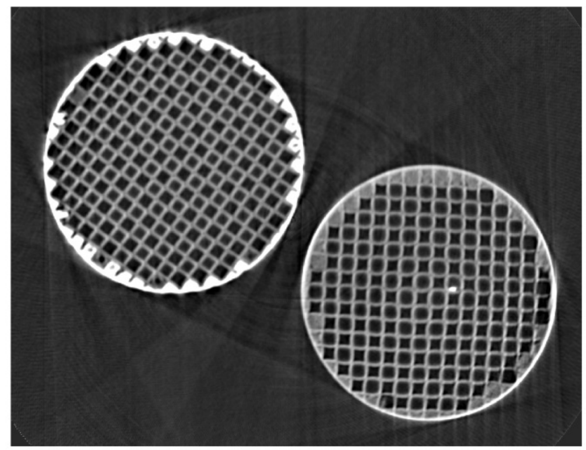
Medical imaging

X-ray CT, SPECT, PET, MRI



Waste/NDT imaging

Neutron CT



- Algebraic, statistical reconstruction algorithms: SIRT, MLEM
- Academic proof-of-principle and commercial/production code
- Participant in ALERT TO3 (recon) , TO4 (ATR development)

Method: SIRT in a Nutshell

SIRT iteratively tries to guess image configuration that best satisfies Ax = b in a weighted least squares sense

- Notation: A=source-to-detector ray model, x=image, b=data
- Least squares corrects large errors at expense of small errors
- Weighting changes importance of individual ray-based errors
- Regularization improves numerical stability, adds smoothing

SIRT is a good alternative to filtered backprojection since it allows modeling of the imaging process (geometry etc)

SIRT uses simultaneous updating of all pixels making it numerically robust with respect to data inconsistencies

Method: SIRT in Equation Form

Method for solving weighted least squares problem:

$$x^* = \text{argmin } \|Ax - b\|_R^2$$

Solution computed iteratively given initial estimate:

$$x^{(k+1)} = x^{(k)} + \alpha\, CA^T R\, (b - Ax^{(k)})$$

Matrix A: scanner geometry (ray-pixel area intersection)
Vector b: log-normalized projection data (fan-beam)
Vector x: reconstructed image (range: -100:30,000 HU)

Matrix R: solution weight matrix (all RA row sums = 1)
Matrix C: preconditioning matrix (all CA^T row sums = 1)

Scalar α: relaxation param. (near-optimal value = 1.99)

Wx: Data Weighting

The linear system Ax = b is the discretized version of

$$\int_L x(s)ds = -\ln \frac{I_1}{I_0}$$

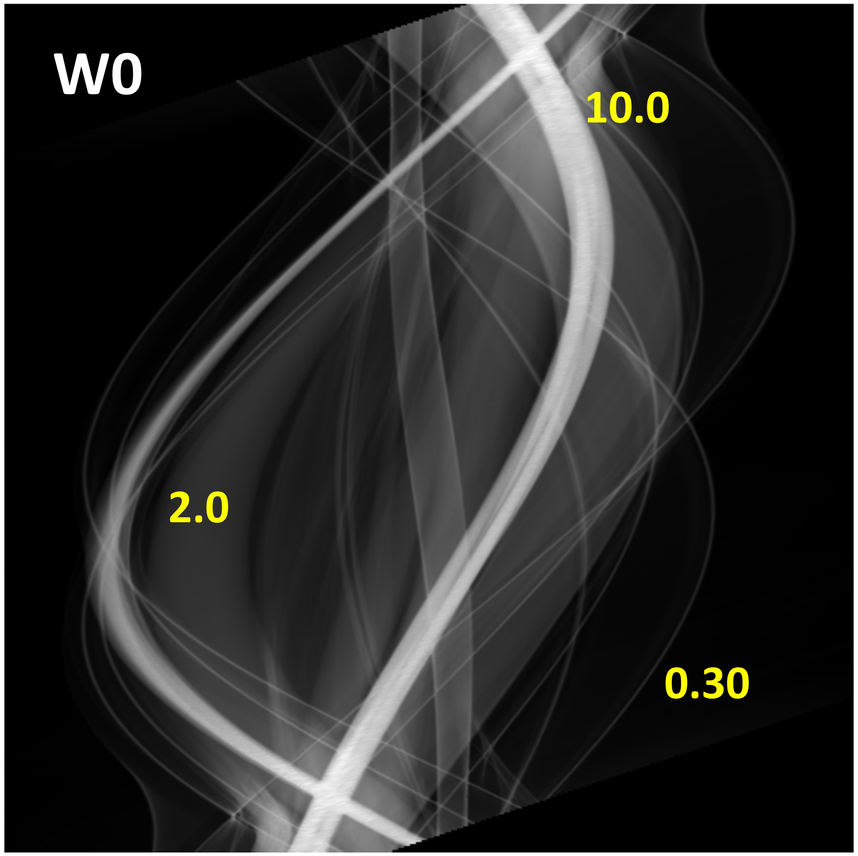
Low SNR data maps to large log-normalized value which plays a dominant role in the least squares computation.

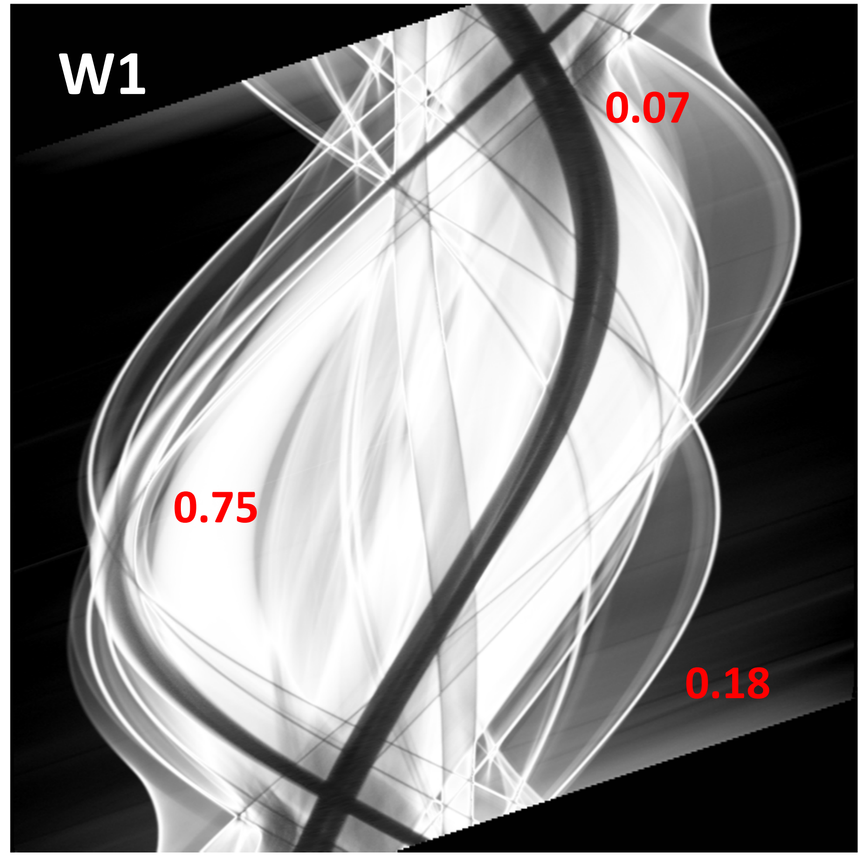
Effect suppressed by weighting of matrix and sinogram:

$$\left.\begin{array}{l} W0: w_i = 1 \\ W1: w_i = e^{-0.5\, b_i} \end{array}\right\} \quad \left\{\begin{array}{l} a_{ij} = w_i\, a_{ij} \\ b_i = w_i b_i \end{array}\right.$$

Matrices R and C are computed for weighted A matrix leading to preservation of known convergence properties

Wx: Data Weighting





Heavily attenuated (bright=low SNR) rays in W0 are mapped to low values in W1 reducing their impact on the reconstruction, thereby improving image quality

Bx: Tikhonov Regularization

SIRT can be made less sensitive to small changes in the input through application of Tikhonov regularization:

$$x_* = \text{argmin} \; \|Ax - b\|_R^2 + \beta \|x\|_2^2$$

Solution computed with preference towards smaller norm:

$$x^{(k+1)} = (I - \alpha\beta C)\, x^{(k)} + \alpha\, CA^T R\, (b - Ax^{(k)})$$

Added benefit: spatially variant smoothing of image

Parameter β controls data fit versus min-norm trade-off

Bx means β = 0.0x. Thus, B0 means no regularization while B1 thru B5 means light thru heavier regularization

Conjugate Gradient Framework

Method for solving symmetric pos. definite linear system:

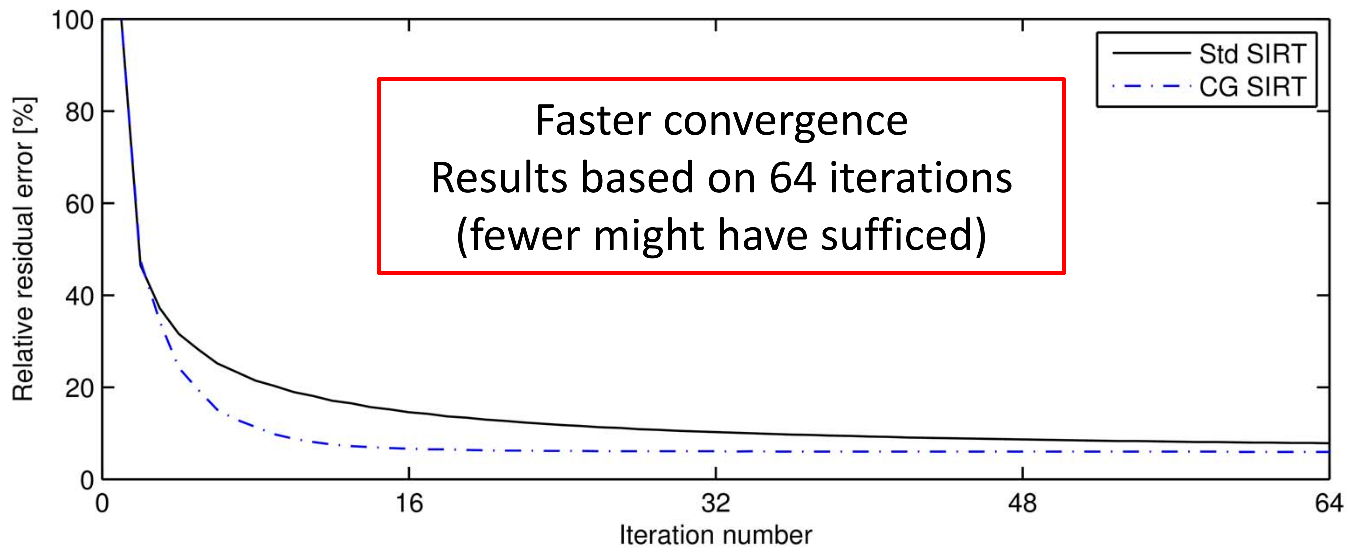
$$z^* = \operatorname{argmin} \|Mz - h\|_2^2 + \beta \|z\|_2^2$$

Map original SIRT problem to above L2 version

$$m_{ij} = a_{ij}/\sqrt{r_i c_j},\ h_i = b_i/\sqrt{r_i},\ z_j = x_j\sqrt{c_j}$$

Remap solution

$$x_j^* = z_j^*/\sqrt{c_j}$$



Faster convergence
Results based on 64 iterations
(fewer might have sufficed)

Computational Cost

- Problem size ………………………………………………

  Projection: 864 views x 888 rays (corner filling discarded)
  Image: 512 x 512 pixels (inscribed circle constraint)
  System matrix: 950M non-zero elements (single-precision float)

- Implementation ………………………………………

  Code written in C using POSIX based multi-threading

- Computer platform ……………………………………..

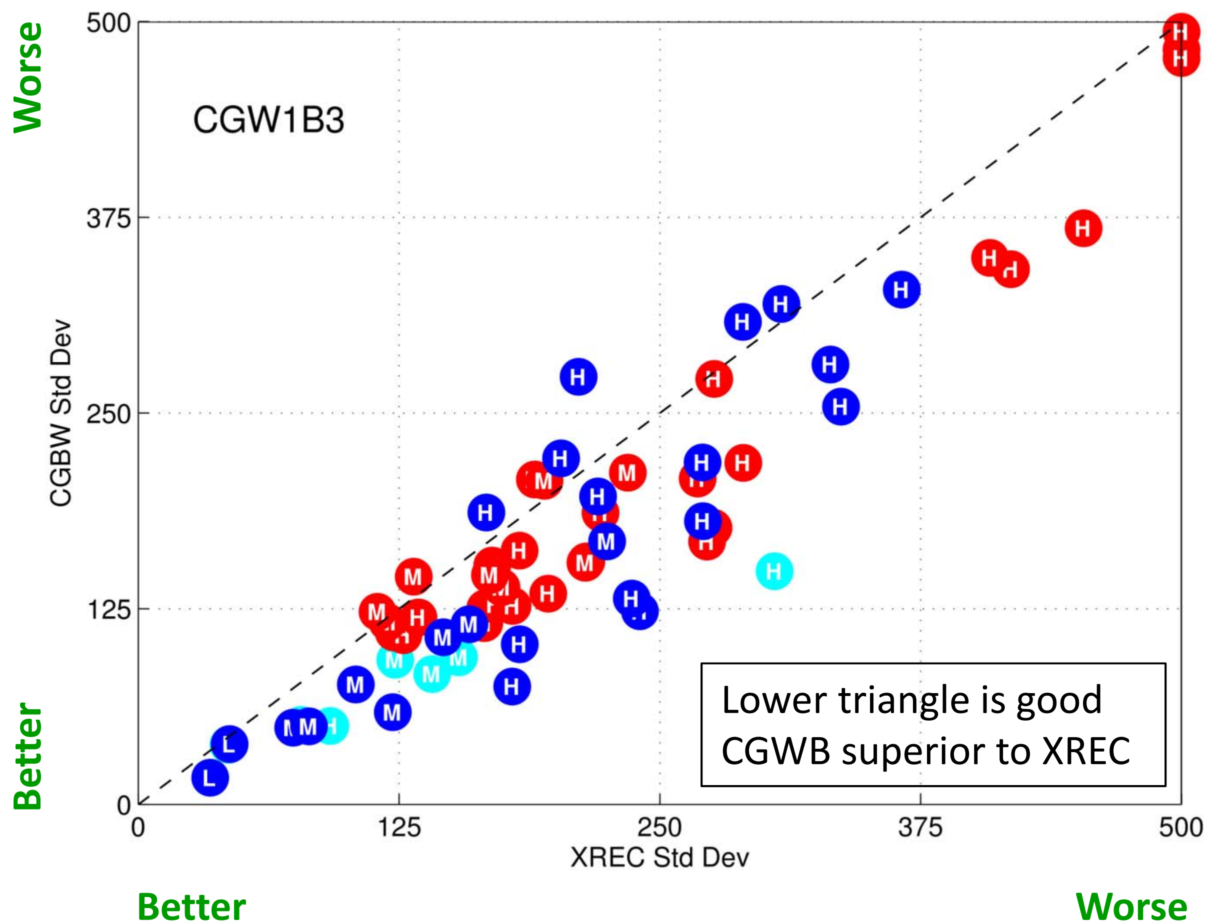
  Dell Precision PC w/dual quad-core 2.26 GHz Xeon CPUs

- Timing numbers ………………………………………...

  System matrix: 20 sec. (could be stored and read from file)
  SIRT-to-CG: 4 sec. (incl. projection and matrix weighting)
  Per iteration: 2 sec. (incl. active set-like bound constraints)

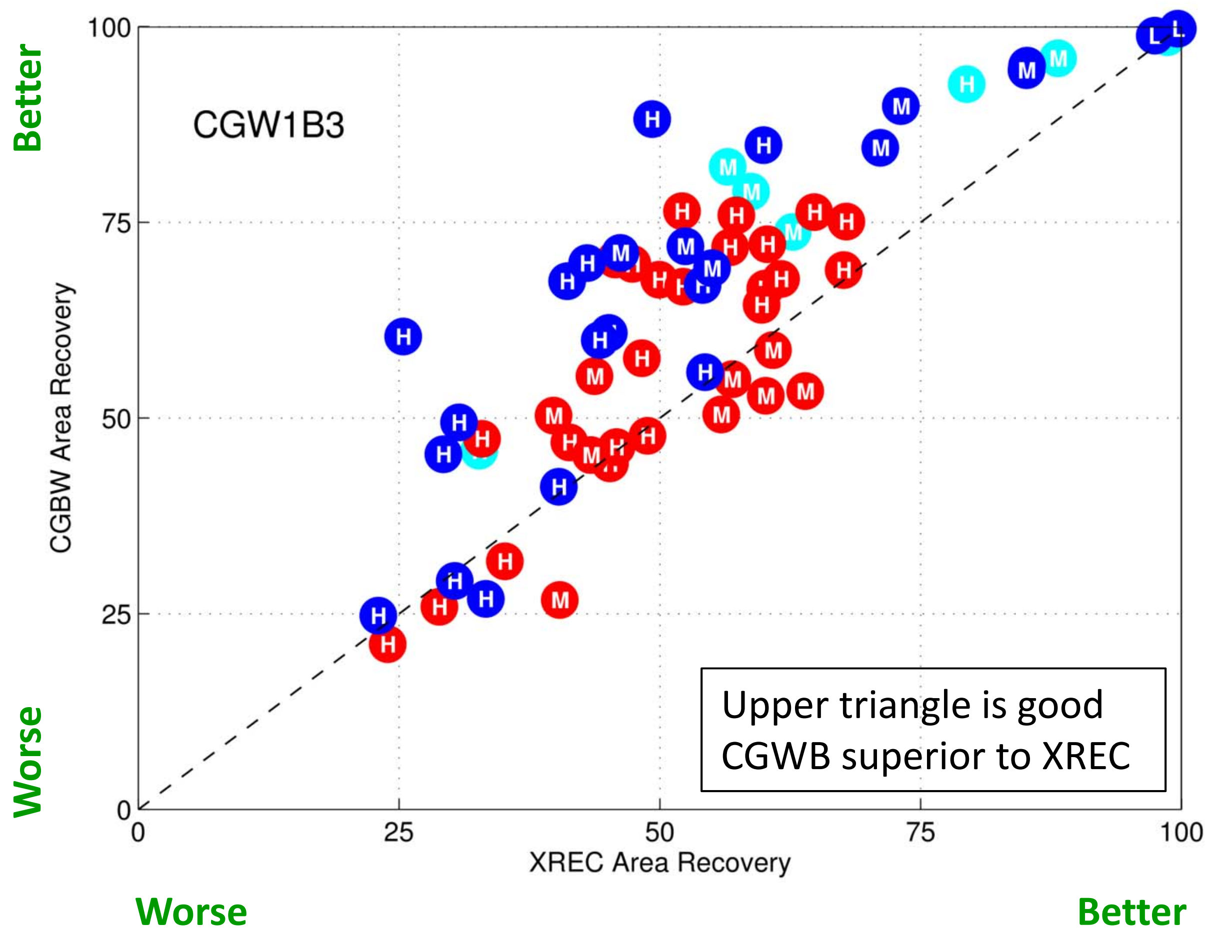
Scatter Plot: Standard Dev

XREC and CGWB results plotted per object



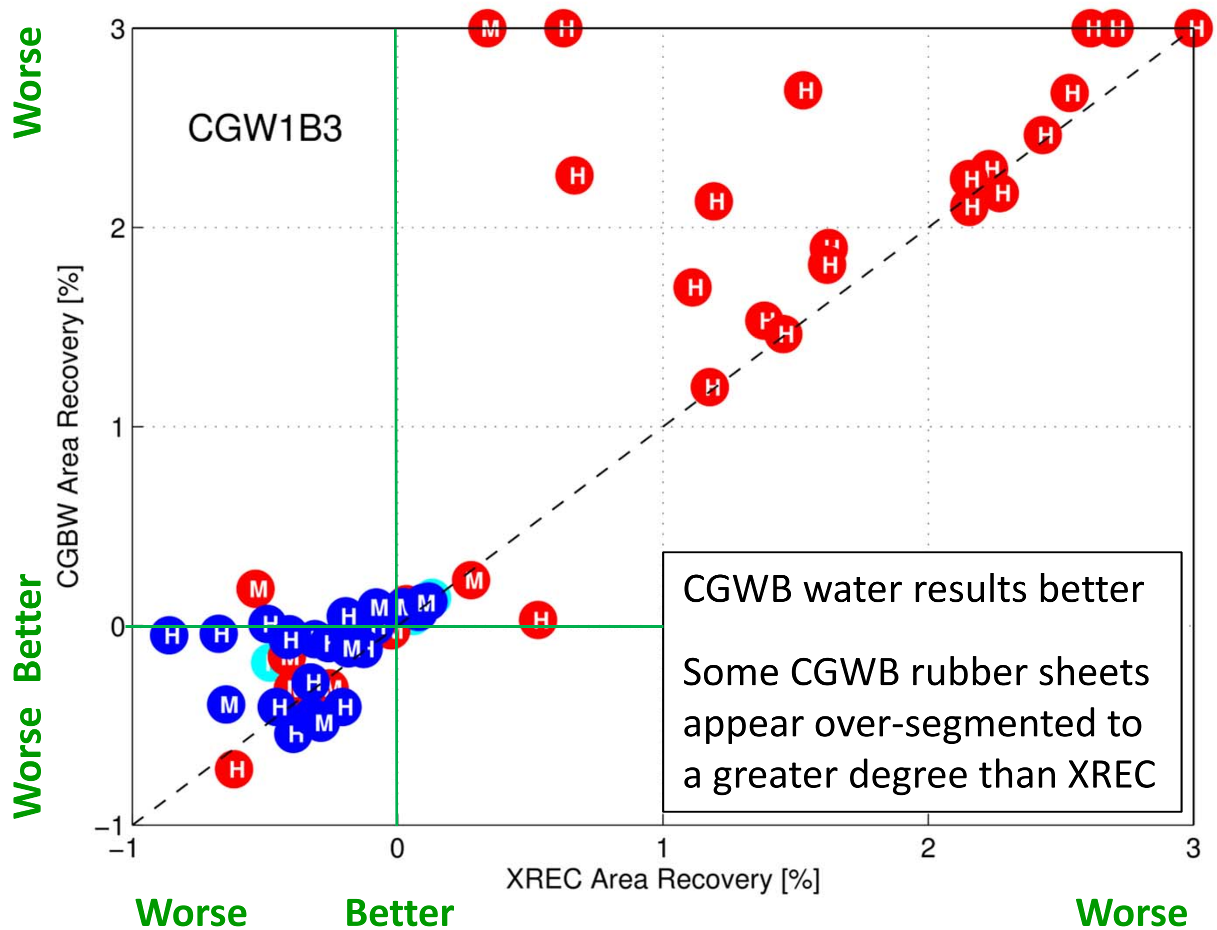
Scatter Plot: Area 1

Num. pixels in median ± 100 HU range relative to ground truth mask size



Scatter Plot: Area 2

Num. pixels in Tumbler mask region relative to ground truth mask size



Example: LLNL TWO Slice 068

No visible artifacts.

No visible improvement.

Example: Med Clutter 1 Slice 235

Some metal artifact streaking.

Slightly smoother image appearance.

Example: Med Clutter 2 Slice 326

Some metal artifact shading. Some streak noise.

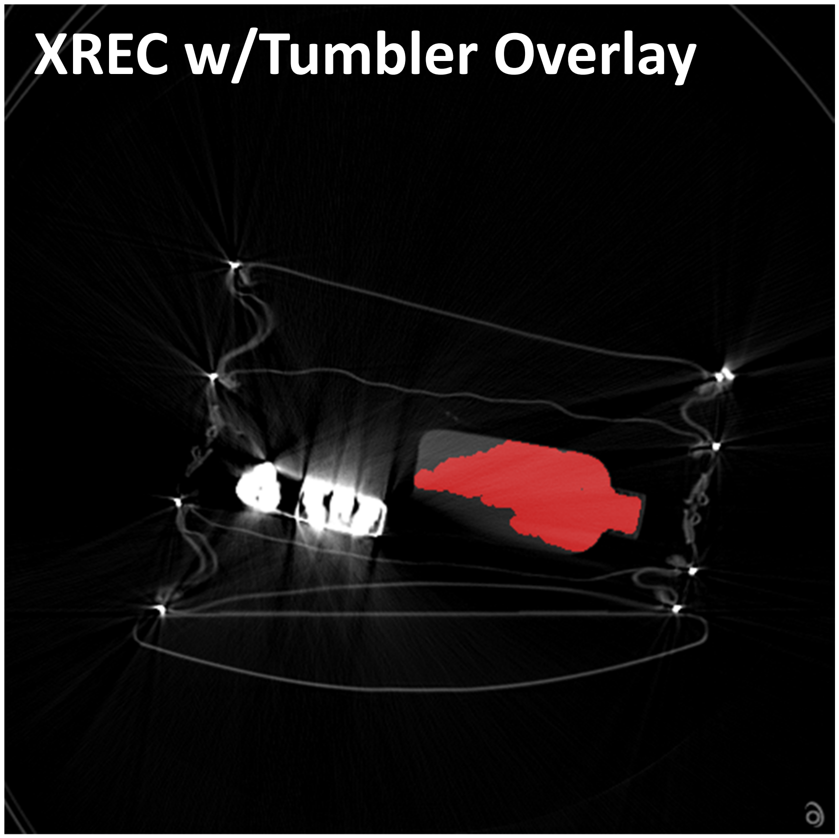
Less metal artifact shading. Less streak noise.

Example: Med Clutter 4 Slice 134

Some metal artifact streaking.

Slightly smoother image appearance.

Example: High Clutter 1 Slice 350



Some metal artifact shading. Some streak noise.

Less metal artifact shading. Less streak noise.

Example: High Clutter 3 Slice 222

Some metal artifact shading. Some streak noise.

Less metal artifact shading. Less streak noise.

Summary of Pros and Cons

SIRT Weaknesses

- Excessive regularization may cause object fusion

- Convergence rate
- Computational cost

SIRT Strengths

- Accurate geometry model
- Flexible data corrections

- Controlled regularization yields smoother regions

- Can be preconditioned
- Can easily be parallelized

Suggestions for Future Work

- Apply CGSIRT to data from security scanner
- Incorporate pre/post reconstruction metal and other artifact reduction schemes
- Study alt. weighting, regularization schemes

- Reduce computational cost: many-core and vector utilization, limit computation to region of interest (e.g., suitcase or object inside).

Acknowledgments

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