Model-Based Iterative Reconstruction for CT Luggage Screening

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ALERT TO#3 Project Review

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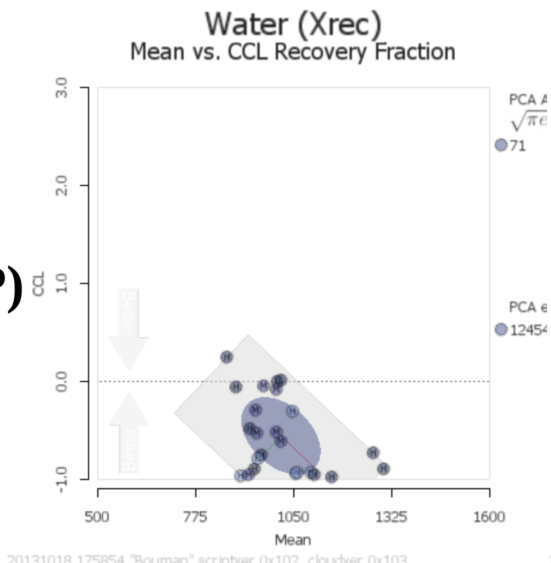
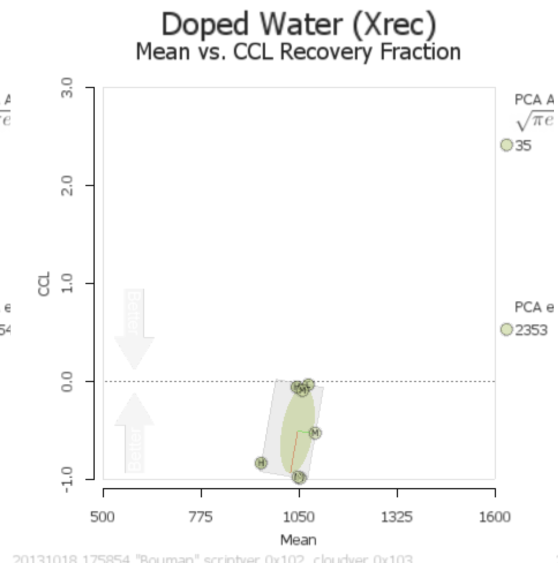
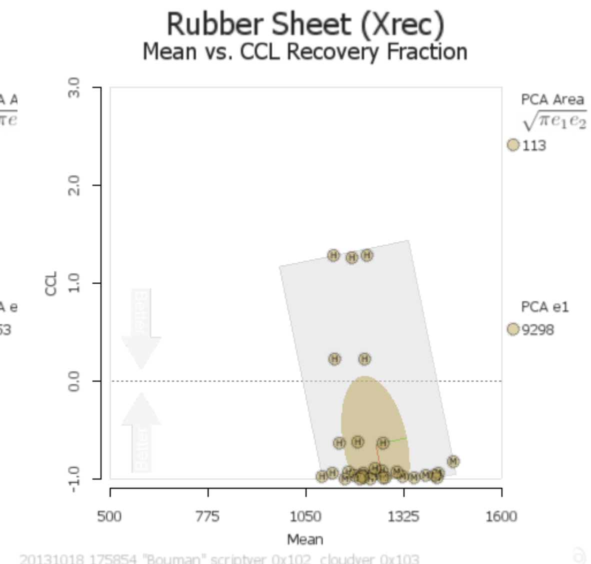
Cloud Plots

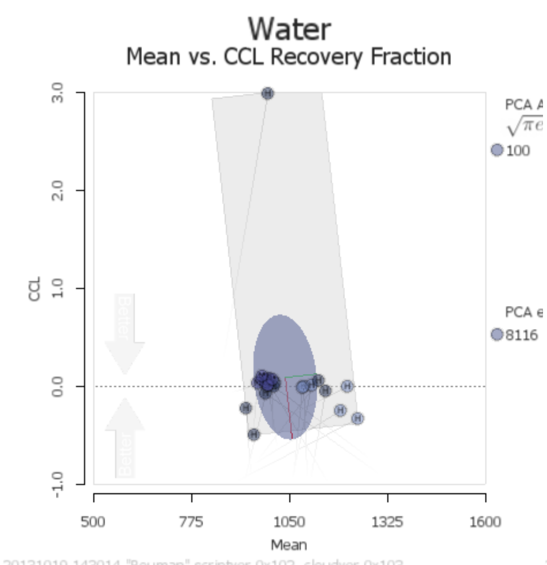
Xrec (FBP)

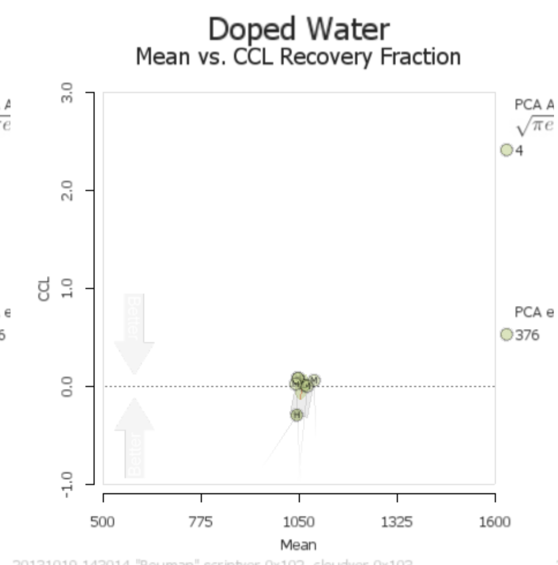


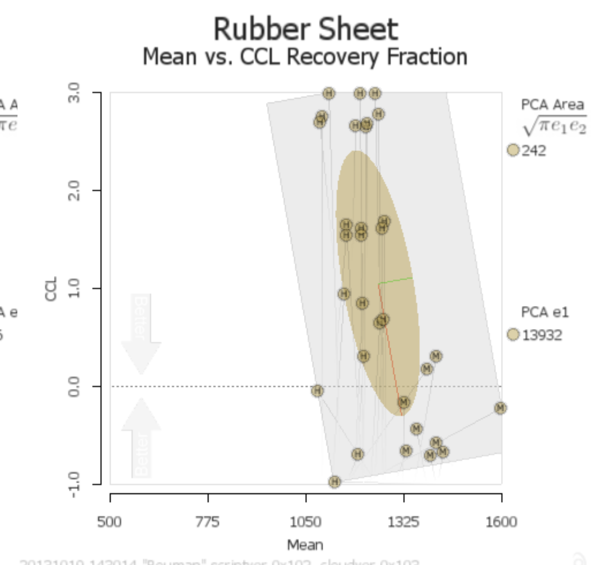




MBIR

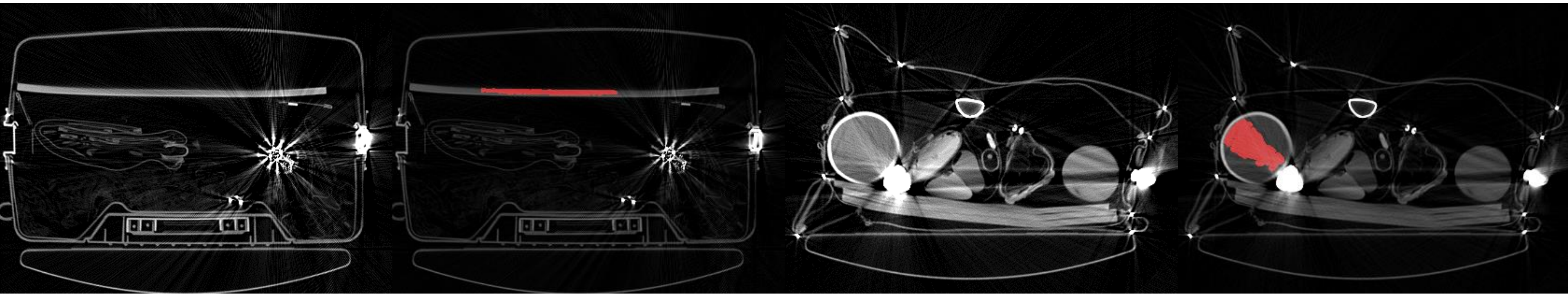




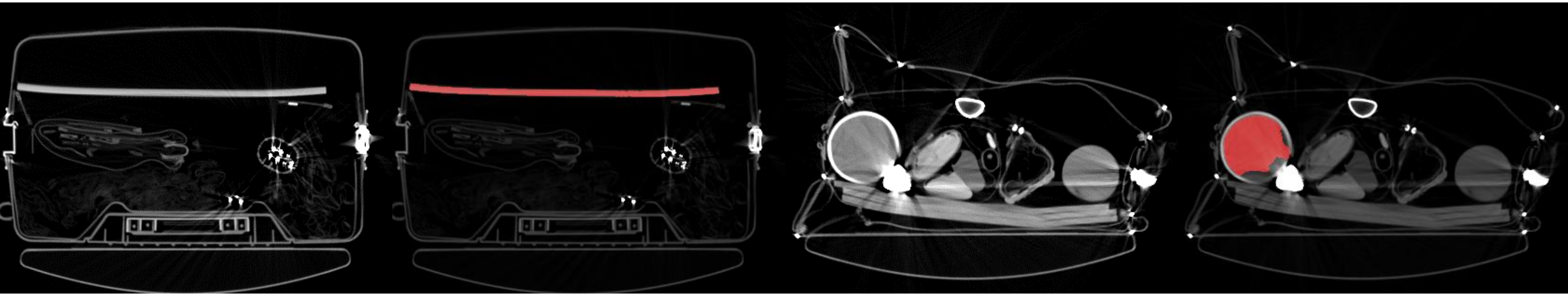


Sample Images

Medium Clutter (1-123)   High Clutter (1-239)



Xrec (FBP), and Tumbler segmentation



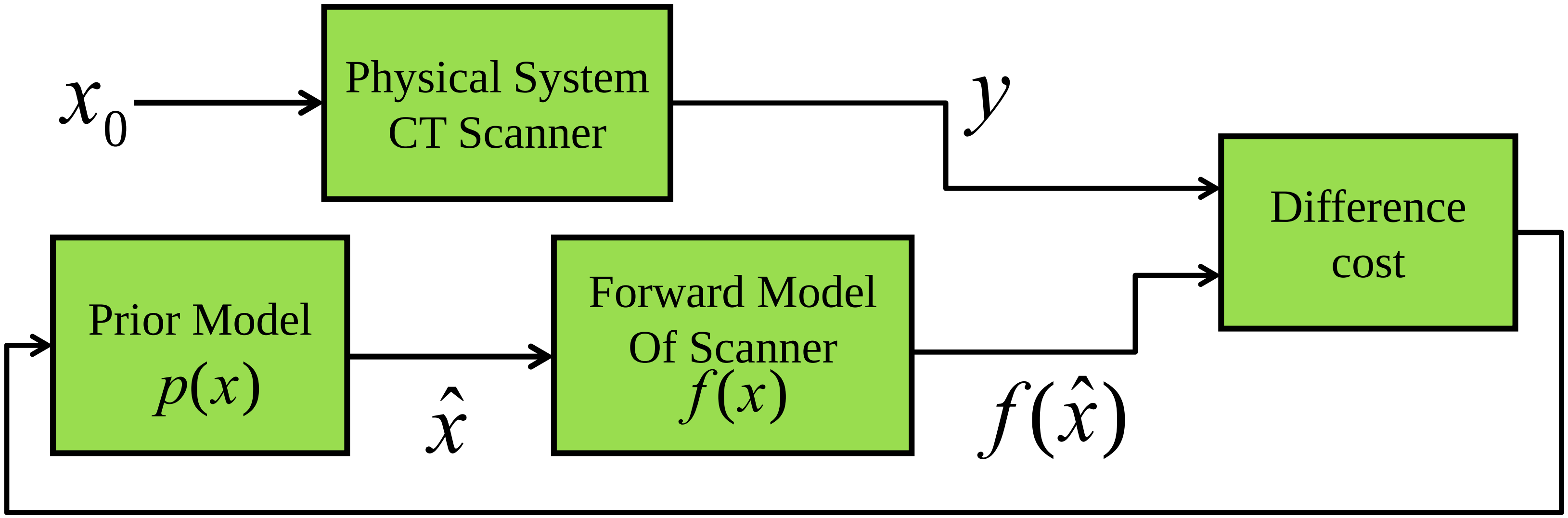
MBIR, and Tumbler segmentation

Introduction to the institution and researchers

- Pengchong Jin – 5th year Ph.D. student, Purdue University. B.Eng, ECE from HKUST (2009). Statistical signal processing, inverse problems
- Charles Bouman – Showalter Professor of ECE, Purdue University. BSEE U. Penn, Ph.D. Princeton (1989). Stochastic image modeling, image rendering, tomography
- Ken Sauer – Assoc. Prof. of EE, University of Notre Dame. BSEE Purdue, Ph.D. Princeton (1989). Statistical methods in tomographic inverse problems, optimization

Methods

Model-based Iterative Reconstruction



- $x \in R^N$: image to be estimated, $y \in R^M$: sinogram
- Invert by computing the MAP estimate

$$\hat{x}_{MAP} = \arg\max\{\log p(y|x) + \log p(x)\}$$

Likelihood function
Forward model derived from system physics
Measurement noise modeling

Prior
Regularization
smoothing

Model-Based Iterative Reconstruction

- Cost function for image reconstruction

$$\hat{x}_{MAP} = \arg\min_{x \geq 0} \left\{ \frac{1}{2} \| y - E[y \mid x] \|_W^2 + \sum_{\{s,r\} \in C} \alpha_{s,r} \rho(x_s - x_r) \right\}$$

  - $x \in R^N$: image to be estimated
  - $y \in R^M$: sinogram, measurements
- Data fit term of cost
  - Accurate model of X-ray forward projection $E[y \mid x]$
  - Accurate noise model, weighting matrix W

Main idea: Adapt data penalty term to take "pressure" off metal-corrupted measurements

Approach 1: Better Forward Projection Model

- Classic forward model assumes $E[y|x] = Ax$
- Energy-dependent attenuation -> beam "hardening"
- Our approach
  - Separate materials into low and high densities

$$E[y|x] = h(p_{L,i}, p_{H,i}) = \sum_k \sum_l g_{k,l} (p_{L,i})^k (p_{H,i})^l$$

  - Two separate "material" projections

$$p_{L,i} = \sum_{j=1}^{N} A_{i,j} x_j (1 - b_j), \quad p_{H,i} = \sum_{j=1}^{N} A_{i,j} x_j b_j$$

  - $b_j \in \{0, 1\}$: indicator of the j-th pixel for high density

Approach 1: Joint Estimation and Correction

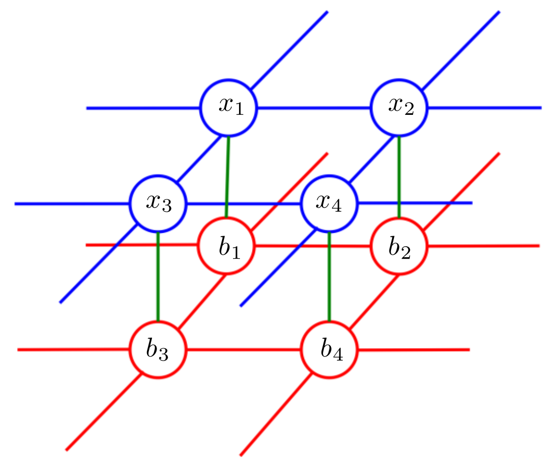
- Joint optimization problem

$$\{\hat{x}, \hat{b}, \gamma\} = \arg\min_{x \geq 0, b, \gamma} \left\{ \frac{1}{2} \sum_{i=1}^{M} \left(y_i - \sum_k \sum_l \gamma_{k,l} (p_{L,i})^k (p_{H,i})^l \right)^2 + U(x,b) \right\}$$

  - Simultaneous image reconstruction and beam hardening correction as a joint optimization problem
  - Use alternating optimization for x, b and $\gamma_{k,l}$s.

- Joint regularization scheme $U(x,b)$

$$U(x,b) = \sum_{\{s,r\} \in C} \alpha_{s,r} \rho(x_s - x_r) + \sum_{\{s,r\} \in C} \eta_{s,r} \delta(b_s \neq b_r)$$
$$+ \beta \sum_{j=1}^{N} (x_j - T)_+ (1 - b_j) + (T - x_j)_+ b_j$$



Approach 2: Better Noise Model (Weighting)

- A typical weighting from Poisson-Gaussian model*

$$w_i = \frac{\lambda_i^2}{\lambda_i + \sigma_e^2} \propto \frac{e^{-2y_i}}{e^{-y_i} + C_e}$$

  - Large dynamic range when metals are present
- Novel weighting scheme

$$w_i = I_i e^{-y_i} + (1 - I_i) e^{-y_i/2}$$

  - $0 \le I_j \le 1$: fraction of contribution from high density material
  - Calculated using the initial image

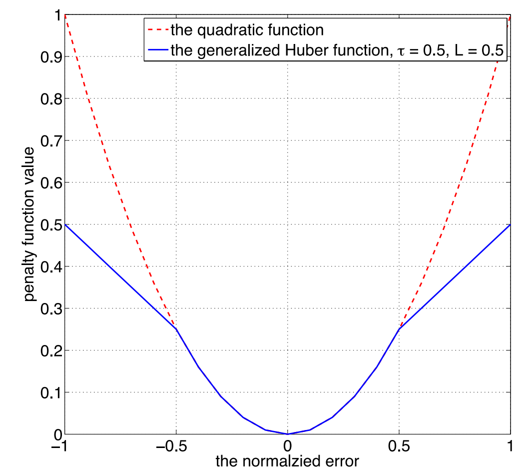
*Sauer, Bouman, TIP, Feb. 1993, Thibault, Sauer, Bouman, Hsieh, Medical Physics, Nov. 2007

Approach 3: Robust treatment of outliers

- The actual measurement can differ from the physical model significantly
  - Due to various effects coupled together, beam hardening, scattering, metal partial volume, etc.
  - Hard to integrate them individually
- Consider a modified model to reduce the influence of the defective measurement to the MBIR cost*

$$-\log p(y \mid x) = \frac{1}{2} \sum_{i=1}^{M} H_{L,\tau}\left(\sqrt{w_i}\left(y_i - A_{i,*}x\right)\right)$$

  - $H_{L,\tau}(\cdot)$ is the generalized Huber function



*Venkatakrishnan, Drummy, Graef, Simmons, Bouman, EI, 2013

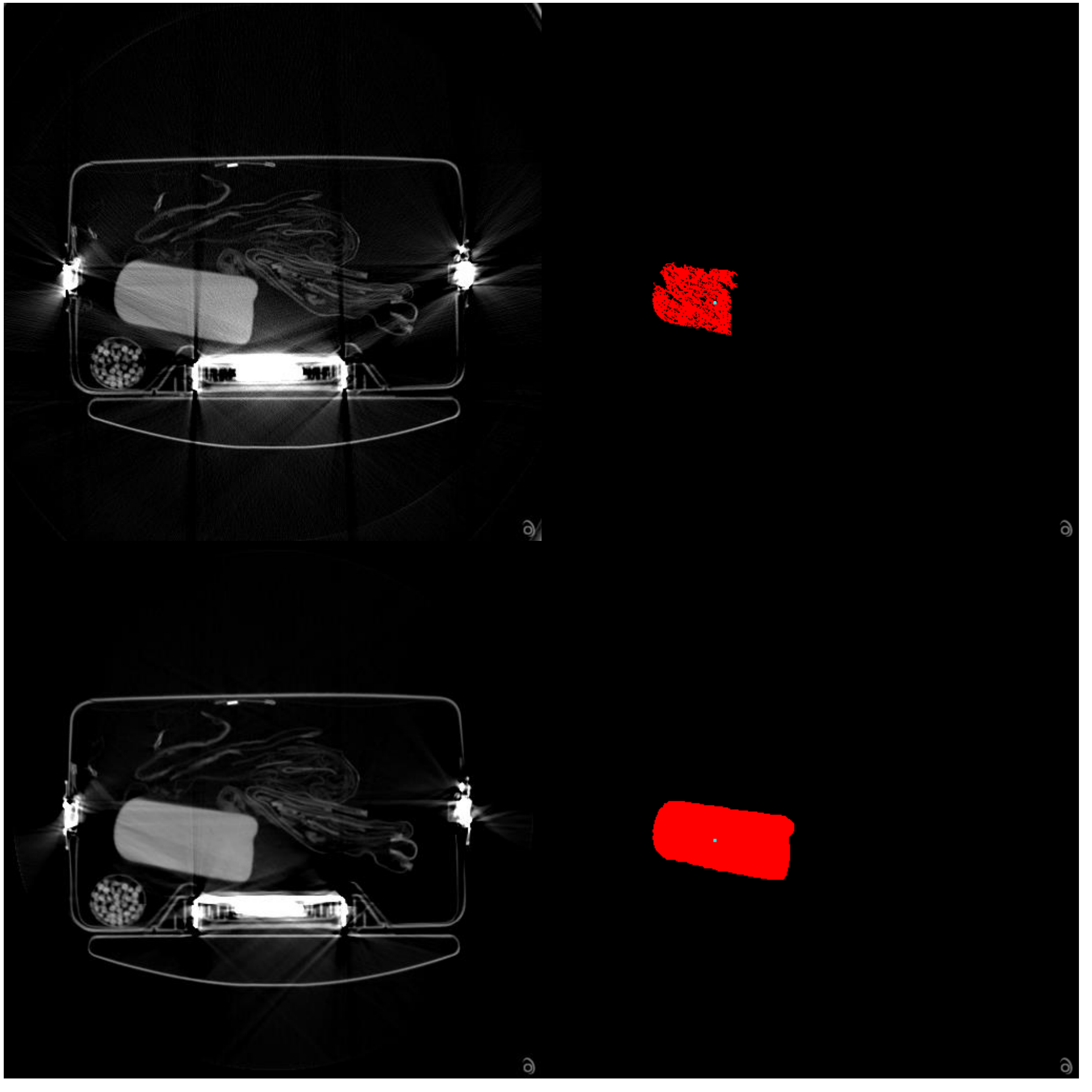
Results

Doped Water Recon/Segmentation

Medium Clutter (2-326) CCL results

Xrec (FBP)

MBIR

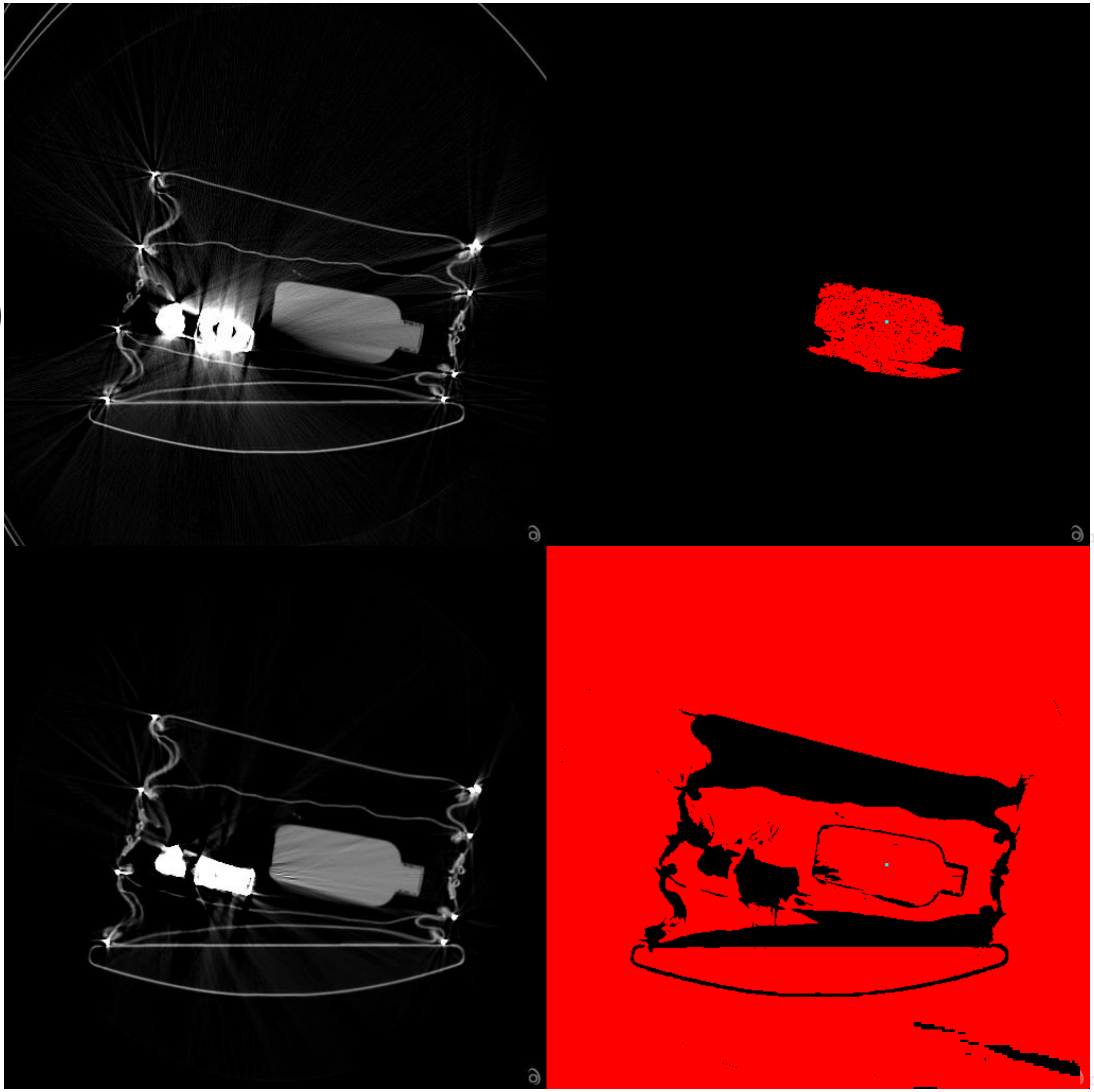


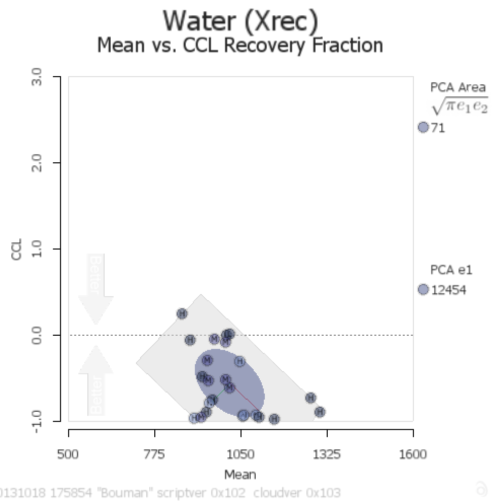
MBIR/CCL Epic Fail

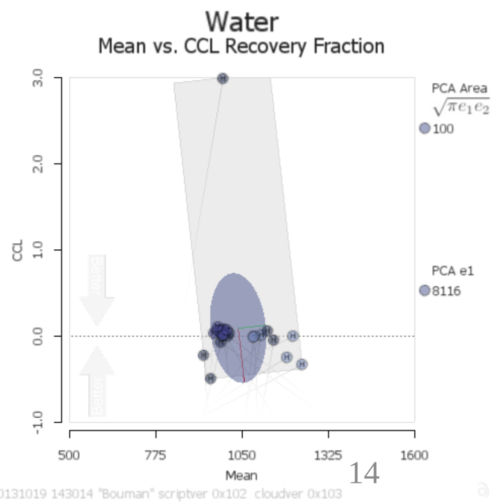
High Clutter Water (1-350) CCL results

Xrec (FBP)

MBIR





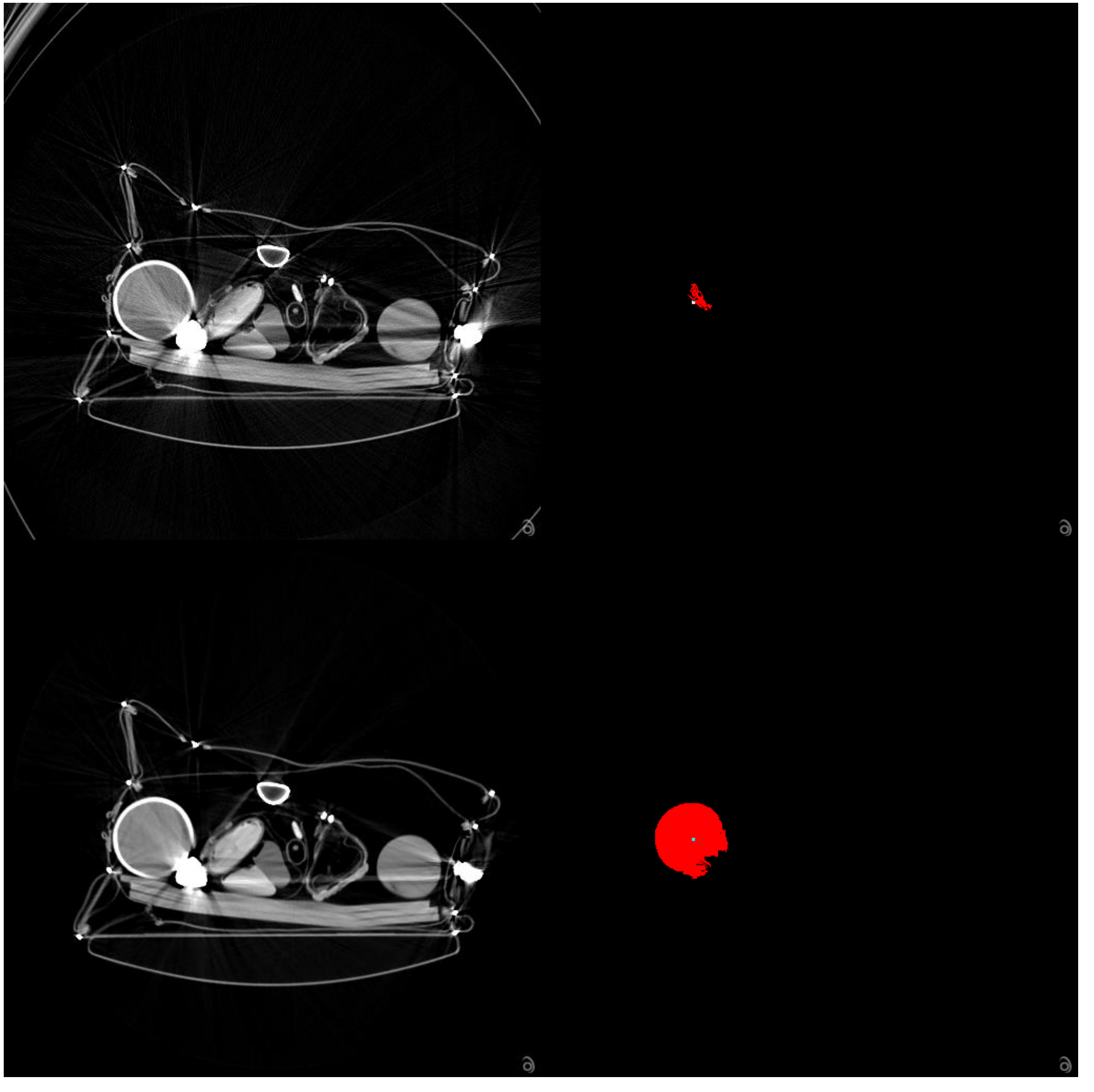


Sample Water Recon/Segmentation

High Clutter (1-239) CCL results

Xrec (FBP)

MBIR

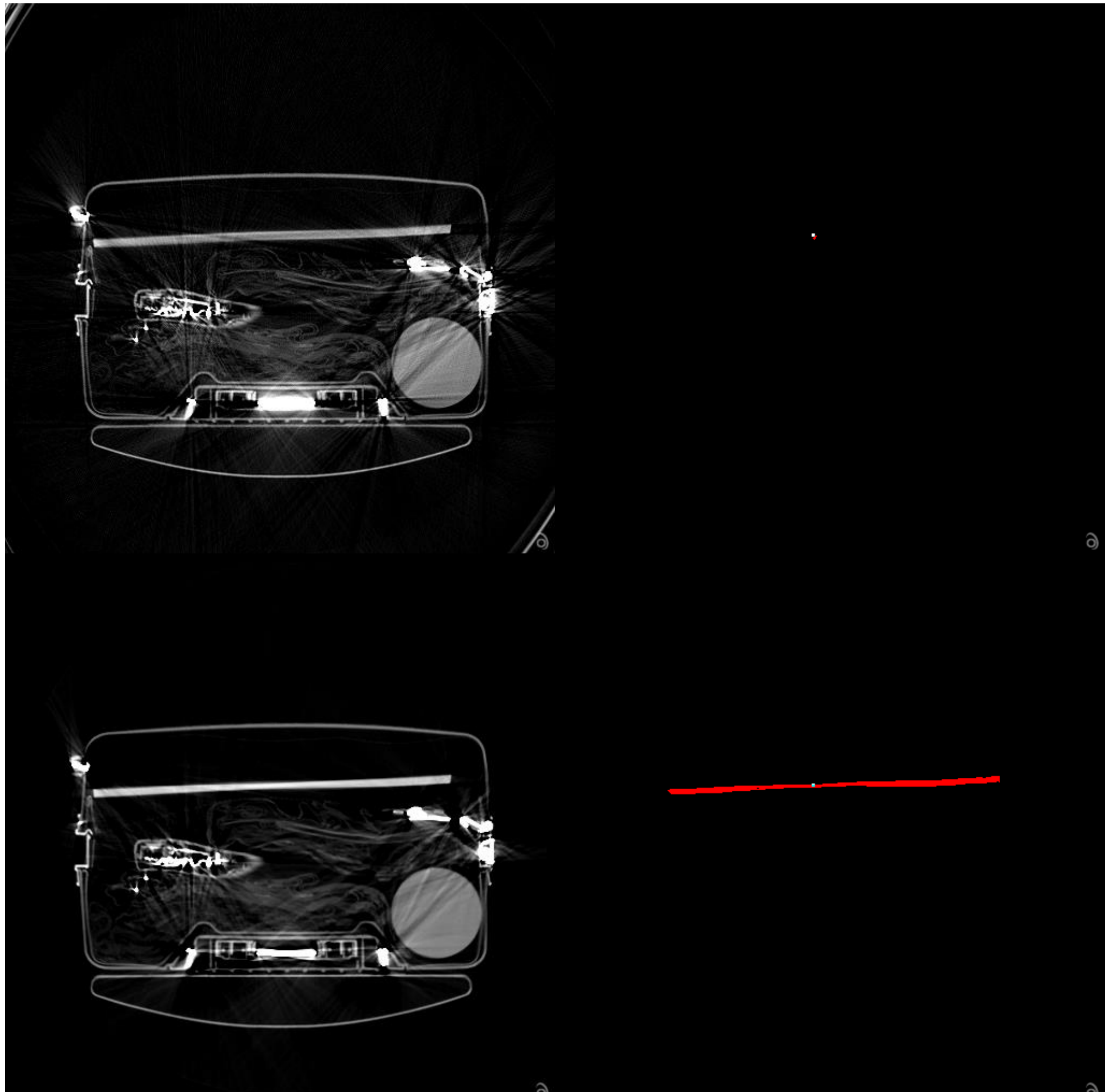


Sample Sheet Recon/Segmentation

Medium Clutter (1-281)　　CCL results

Xrec (FBP)

MBIR

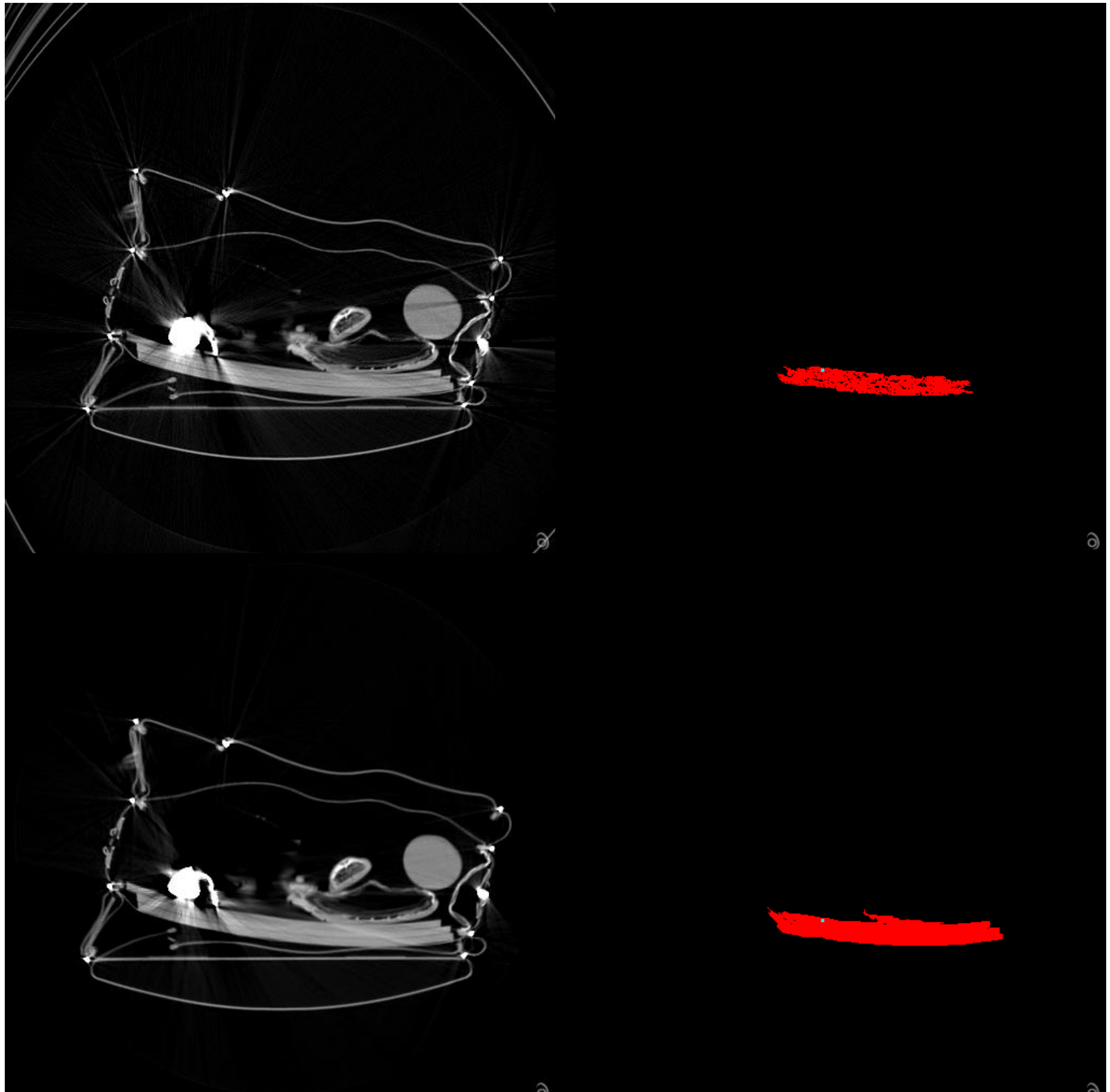


Sample Sheet Recon/Segmentation

High Clutter (1-299) CCL results

Xrec (FBP)

MBIR



Summary

- Positives of MBIR in TO3:
  - Good suppression of noise in bulk materials
  - Options for reduction of metal artifacts to improve segmentation
  - Improved resolution
- Downsides
  - Don't yet see "magic bullet" for metal
  - Key materials/configuration (rubber sheets + metal) remains challenging
  - Huge computational cost is barrier to entry
  - Rebinning for speed, simplicity may cost resolution available in accurate system modeling

Quo Vadis?

- More aggressive treatment of metal
  - ✓ Formulate fixed correction function for beam hardening
  - ✓ Projection replacement algorithms
- Resolution enhancement through modeling of rebinning losses
  - ✓ Expand detector in forward model
  - ✓ High frequency pre-emphasis of sinograms
- Improve *a priori* image model
  - ✓ Take advantage of 3^{rd} spatial dimension
  - ✓ Tailor prior to discrete-valued materials
    - o Total variations-like?

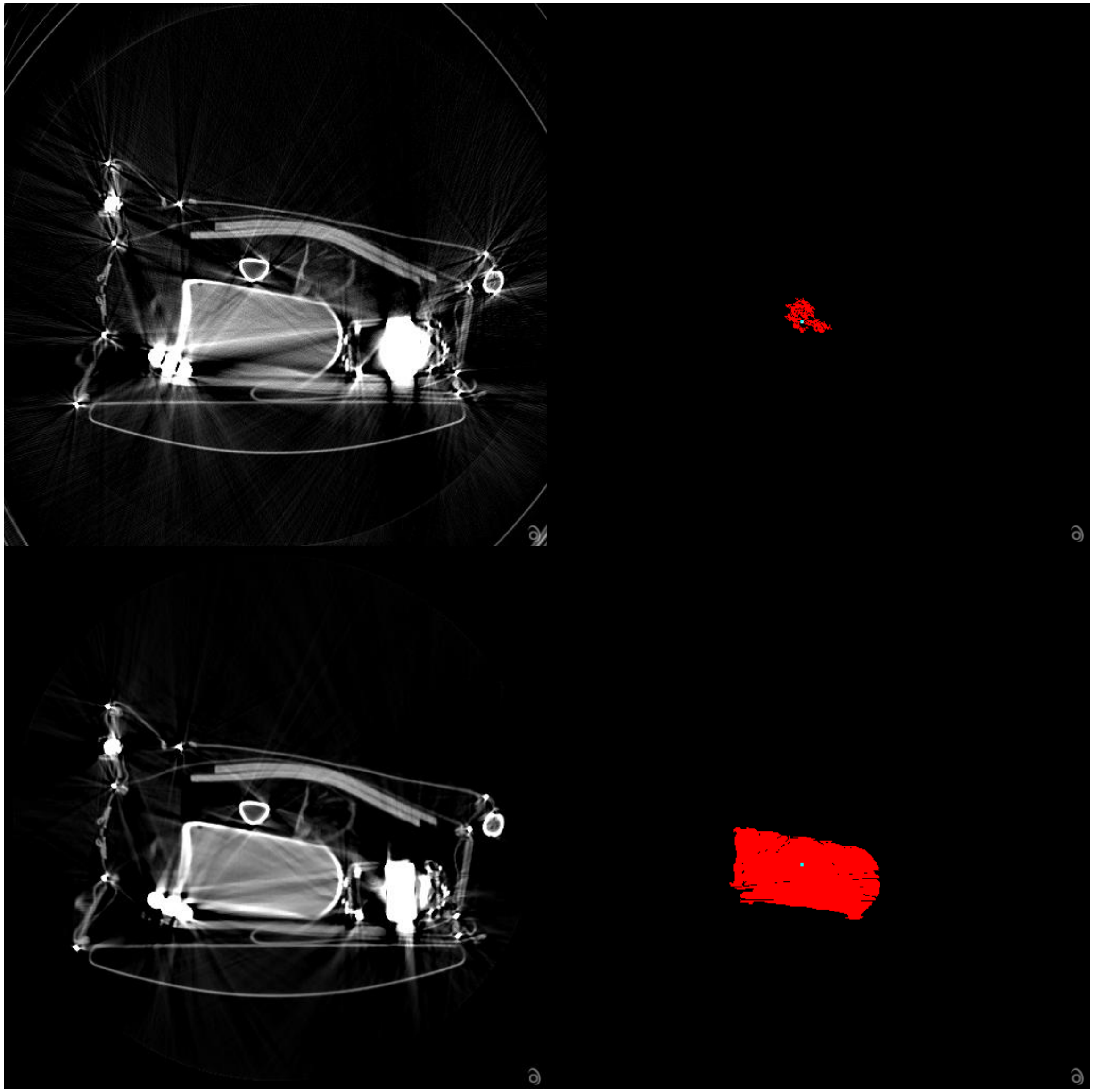
Backup Slides

Metal Artifact Reduction

High Clutter (3-194) CCL results

Xrec (FBP)

MBIR



Model-Based Iterative Reconstruction

- Statistical model for image reconstruction

$$\hat{x}_{MAP} = \arg\max_{x \geq 0} \left\{ \log p(y \mid x) + \log p(x) \right\}$$

$$= \arg\min_{x \geq 0} \left\{ \frac{1}{2} \left\| y - E[y \mid x] \right\|_W^2 + \sum_{\{s,r\} \in C} \alpha_{s,r} \rho(x_s - x_r) \right\}$$

  - $x \in R^N$: image to be estimated
  - $y \in R^M$: sinogram, measurement
- Forward model $p(y \mid x)$
  - Accurate model of X-ray forward projection $E[y \mid x]$
  - Accurate noise model, weighting matrix W
- Image prior model $p(x)$
  - Regularize undesired image behavior, smoothing

Approach 1: Better Forward Projection Model

- Classic forward model assumes

$$E[y \mid x] = Ax$$

  - $A \in R^{M \times N}$: linear forward projection operator
  - Energy-dependent attenuation, broadness of X-ray spectrum
  - Beam hardening effect, nonlinear relationship

- Our approach
  - Different materials can be separated by their densities

$$E[y \mid x] = h(p_{L,i}, p_{H,i}) = \sum_k \sum_l g_{k,l} (p_{L,i})^k (p_{H,i})^l$$

  - Two separate "material" projections

$$p_{L,i} = \sum_{j=1}^{N} A_{i,j} x_j (1 - b_j), \quad p_{H,i} = \sum_{j=1}^{N} A_{i,j} x_j b_j$$

  - $b_j \in \{0, 1\}$: indicator of the j-th pixel that of high density

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$$\{\hat{x}, \hat{b}, \gamma\} = \arg\min_{x\geq 0, b, \gamma} \left\{ \frac{1}{2} \sum_{i=1}^{M} \left(y_i - \sum_k \sum_l \gamma_{k,l} (p_{L,i})^k (p_{H,i})^l \right)^2 + U(x,b) \right\}$$

  – Simultaneous image reconstruction and beam hardening correction as a joint optimization problem
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- Design joint regularization scheme $U(x,b)$

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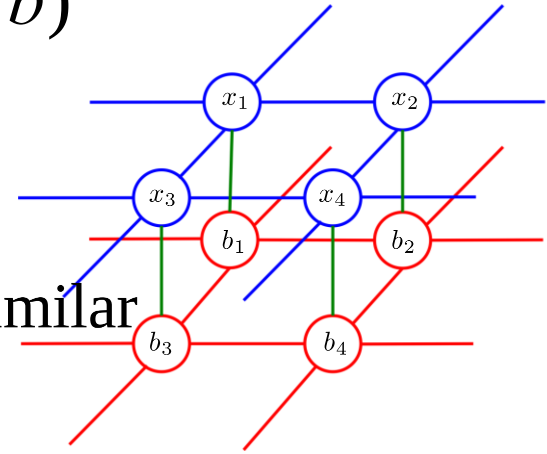


  – Want neighboring pixels, and labels to be similar
  – Want pixels and labels to be consistent
  – Use $T = 2000$ HU

Approach 2: Better Noise Model (Weighting)

- A typical weighting from Poisson-Gaussian model*

$$w_i = \frac{\lambda_i^2}{\lambda_i + \sigma_e^2} \propto \frac{e^{-2y_i}}{e^{-y_i} + C_e}$$

  – Uniform weighting scheme
  – Large dynamic range when metals are present

- Novel weighting scheme

$$w_i = I_i e^{-y_i} + (1 - I_i) e^{-y_i/2}$$

  – $0 \le I_j \le 1$: “percent” of contribution from high density material
  – Calculated using the initial image

$$I_i = \frac{y_{metal,i}}{y_i} = \frac{\sum_{j=1}^{N} A_{i,j} x_j^{(0)} \delta(x_j^{(0)} > T)}{y_i}$$

*Sauer, Bouman, TIP, Feb. 1993, Thibault, Sauer, Bouman, Hsieh, Medical Physics, Nov. 2007

Approach 3: Bad Measurement Rejection

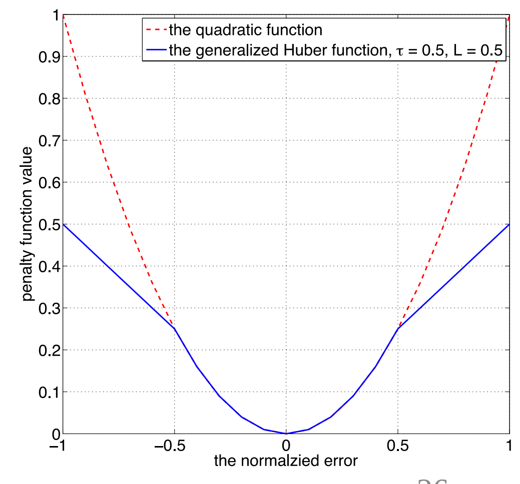
- The actual measurement can differ from the physical model significantly
  - Due to various effects coupled together, beam hardening, scattering, metal partial volume, etc.
  - Hard to integrate them individually
- Consider a modified model to reduce the influence of the defective measurement to the MBIR cost*

$$-\log p(y|x) = \frac{1}{2}\sum_{i=1}^{M} H_{L,t}\left(\sqrt{w_i}\left(y_i - A_{i,*}x\right)\right)$$

  - $H_{L,t}(\cdot)$ is the generalized Huber function

$$H_{L,t}(e) = \begin{cases} e^2 & |e| < L \\ 2tL|e| + L^2(1-2t) & |e| \geq L \end{cases}$$

  - Use $t = 0.5$, $L = 0.5$



*Venkatakrishnan, Drummy, Graef, Simmons, Bouman, EI, 2013